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LIABILITY-DRIVEN INVESTING FOR LIFE INSURERS

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ABSTRACT

Liability-driven investing (LDI) has recently emerged as a powerful paradigm in financial risk management. The basic idea behind LDI is to split the company's balance sheet into two separate balance sheets: one for the liabilities and the matching assets and one for the other (return) assets and the surplus. We show that constructing a proper liability-hedging portfolio (LHP) is very attractive for life insurers because the liability-driven risks can be suppressed without a negative impact on overall return. When these risks are covered by the LHP, the return assets can be optimized using well-known (Markowitz) optimization techniques or (equity) hedge strategies. The LDI approach thus stimulates insurers to address all risks embedded in the insurance liabilities and facilitates the subsequent optimization of the return assets.

Keywords: Liability-driven investing, portfolio construction, life insurers, embedded options

JEL Classification: C15, C88, G11, G22, G32

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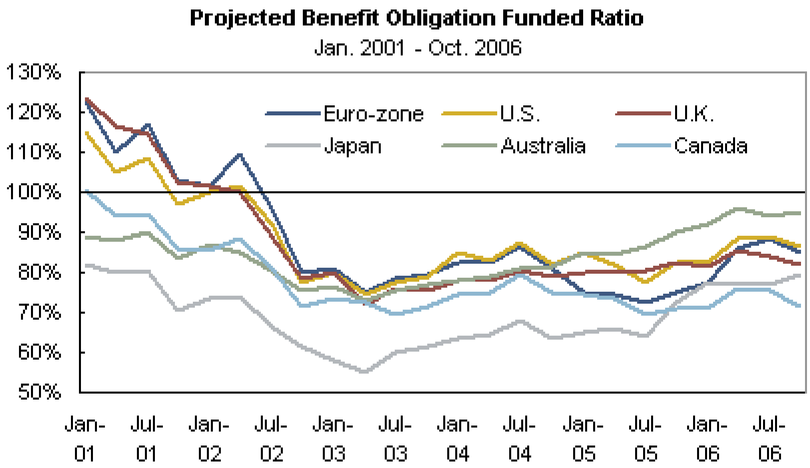
1 Introduction

Liability-driven investing (LDI) is characterized by a central role of the liabilities in the asset allocation process. LDI solutions have been proposed by investment banks and asset managers after many pension funds (as well as other financial institutions) were severely hit by falling interest rates and dropping stock prices in the 2001-2003 period.[4] Figure 1.1 shows that this "financial storm" had a strongly negative impact on the pension's projected benefit obligation (PBO) funding ratio.

Figure 1.1: Funded ratios of pension funds (2001-2006).

Falling interest rates and dropping stock prices during 2001-2003 had severe, negative influences on pension fund PBO's all over the world (data from Towers Perrin).



As a consequence of the huge impact of financial market conditions on key institutions like pension funds, insurers and banks, regulating authorities have tightened the solvency requirements and accounting standards. The following elements are crucial in this regulatory reform:

- Total balance sheet approach (i.e., assets and liabilities);
- Economic or market value;
- Value at Risk (VaR) approach to determine the required capital;
- Wide range of risks (market, insurance, operational, …);
- Capital requirements based on a confidence level on a one year basis;
- Standard versus internal models.

Using these elements, several countries have developed their own solvency tests. Such tests typically use pre-described stress scenarios for the different risk factors (e.g. a shock of –40% for stocks and a shift up and down of the interest rate curve). Institutions should

[4] Maton (2007) and PIMCO (2007) also provide an informal discussion of the factors driving institutions toward LDI.

demonstrate that their surplus is sufficiently large to adsorb these shocks and prevent the occurrence of a negative surplus. The combined effect of the different shocks is determined using assumptions about the correlations between the different risk factors. Large institutions are also encouraged to develop their own internal models to determine the required capital.

The Netherlands has been a frontrunner in these developments by adopting the FTK solvency testing framework, which focuses on overall balance sheet risks (including all assets and liabilities).[5] Twin Peaks and ICA (Individual Capital Assessment) in the United Kingdom, SST (Swiss Solvency Test) in Switzerland and the Traffic Light System in Sweden are examples of similar developments in other countries. The upcoming European Solvency II guidelines for insurers, expected to be in place in 2012, will incorporate these local frameworks.[6] Anticipating these new requirements, financial institutions in many European countries have now moved toward liability-driven investment (LDI) solutions. The idea behind these solutions is that the required capital is reduced by properly matching the liabilities, which makes the company less vulnerable in case of adverse economic situations.

The academic literature with respect to the application of LDI is still rather small. Amenc et al. (2006, 2007) apply the LDI concept to insurance companies and private wealth management. They argue that the LDI concept can be applied successfully in this case and give examples of the effectiveness of both static and dynamic LDI strategies. They show that, given a surplus optimization perspective, more efficient asset mixes can be found by introducing a liability-hedging portfolio (LHP) in the menu of asset classes. LDI solutions thus consist of three basic building blocks (cash, LHP, and a performance portfolio), as opposed to the allocation to standard asset classes, as in the context of regular surplus optimization techniques.

Mindlin (2006) discusses the relevance of LDI solutions for pension funds. He argues that the LDI concept is inadequate for open-ended pension plans since matching assets for on-going plans rarely exist. For example, if the matching asset for an on-going plan existed, it would contain bonds indexed to wage inflation with maturities of 50 years or more. Mindlin therefore concludes that the LDI approach is more appropriate for terminated plans or plans for which termination is likely. He also stresses that the impact of other risk factors (contribution risk, solvency risk, accounting risk, etc.) is not weighted appropriately by LDI proponents.

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[5] In Dutch, FTK stands for "Financieel ToetsingsKader" (Financial Assessment Framework).

[6] Similar to Basel II, Solvency II consists of three different pillars. Pillar I concerns the measurement of assets, liabilities and the required capital. This pillar thus focuses on the more quantitative regulatory aspects. Pillar II concerns the supervisory review process and therefore focuses on the more qualitative aspects. Pillar III addresses disclosure requirements: transparency, open information, etc. See the introductory guide to Solvency II by Towers Perrin (2006) for more information. Amenc et al. (2006) explore the effects of Solvency II and IFRS on ALM and asset management in the insurance industry.

Boender (2007) is also critical of a straightforward application of LDI techniques for pension funds. He points out that LDI is not a completely new concept at all: asset and liability management (ALM) for pension funds has given the liabilities a central role a long time ago (see for example Zenios and Ziemba (2006, 2007) or Ziemba and Mulvey (1998) for excellent overviews). Boender also demonstrates, with an example from practice, that most LDI solutions for open-ended pension funds lead to solutions that are too risk-averse and, as a consequence, too expensive. He remarks that it is common practice that, given an agreed-upon asset allocation, sponsors contribute more in case of a decreasing funded ratio. This risk-reducing effect (for the pension plan) is not considered in LDI solutions. As a consequence, an LDI solution may be overly conservative. A second risk-reducing mechanism, also frequently ignored by LDI, is the postponement of indexation in case of low funded ratios. As a third factor, new pension rights may improve a weak funded ratio if the actuarial price is paid for these rights. If these mitigating effects are ignored, the resulting asset allocation may become overly conservative given a certain long-term ambition.

This paper contributes to the existing literature by examining the consequences of LDI for life insurers. Our approach is similar to Boender (2007) in the sense that we use a scenario approach[7] to study the effectiveness of LDI. Our results are, however, much more positive and indicate that the LDI concept has several major advantages for life insurance companies:

- Dividing the assets in a liability-hedging portfolio (LHP) and a return portfolio (RP) creates a natural benchmark for the LHP portfolio (the liabilities).

- When setting up a proper LHP, one has to explicitly address all risks embedded in the insurance liabilities, including the options embedded in these liabilities.

- When the liability-driven risks are covered by the LHP, the resulting return assets can be optimized using well-known (Markowitz) optimization techniques or (equity) hedge strategies.

In one sense, constructing LHP portfolios for life insurers is more straightforward than for pension funds since most claims are in nominal terms. The issue of hedging inflation risk is therefore of less importance (although exceptions exist, for example in case of insured pension contracts with indexation). Complicating issues like the effect of risk sharing with a sponsor and the effect of conditional indexation are also absent or of less importance (since typically no sponsor exists and claims are not adjusted to inflation). Stated differently, for most insurance companies it is clear what the actual liability is and how the market value of this liability should be determined.[8]

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[7] Also known as Monte Carlo simulation of dynamic financial analysis (DFA).

[8] For example, the CRO Forum has recently released a paper concerning the determination of the market value of insurance liabilities. The CRO Forum's position is that all cash flows should be separated into hedgeable and non-hedgeable components and valued using either mark-to-market or

In another sense, LDI portfolios for insurance companies are relatively complex compared to LHP portfolios for pension funds due to options embedded in insurance contracts. We demonstrate in this paper that matching these embedded options with the proper financial instruments (like swaptions, stock options or other hybrid instruments) is possible, although some mismatch risk can of course remain in practice. We also show that overall risk is determined to a large extent by the market value of the matching assets compared to the market value of the liabilities. If these market values are equal, overall risk is minimized. If the volume of the matching assets becomes smaller than the value of the liabilities, overall risk (and return) increases strongly because more leverage is created on the return balance sheet. After one has determined the LHP and the amount of leverage the resulting (return) assets can be optimized using familiar "asset-only" techniques.

The remainder of this paper is organized as follows. We first explain in Section 2 how a simple LDI balance sheet can be constructed by rearranging the assets on a separate matching and return balance sheet. We then investigate in Section 3 how the matching assets (the LHP) can be optimized in such a way that the market value of the liabilities is replicated for a large set of economic scenarios. Using this optimized LHP, we study risk and return on the overall balance sheet in Section 4. Section 5 concludes.

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mark-to-model approaches. Components of the cash flow for which hedging instruments are available in the financial markets should be valued with reference to the prices of those instruments or using the same option pricing techniques and parameters that are used in valuing the hedge portfolio in the financial markets. For components of the cash flow that are subject to non-hedgeable risks (both financial and non-financial) a mark-to-model adjustment (the market value margin) should be added to the best estimate value of the cash flow. A similar position is found in the latest Solvency II guidelines (see CHEIOPS, 2007).

2 Construction LDI balance sheet

2.1 Example balance sheet

The fair value balance sheet in Figure 2.1 will serve as an example in this paper.

Figure 2.1: The fair value balance sheet serving as an example in this paper.

ASSETS		LIABILITIES	
Stocks	285	**Surplus**	185
Fixed income	665		
		Fair value reserve	
		- Basic reserve	748
		- Profit sharing options	18
		- Unit linked guarantee options	45
		- Present value of fees for unit linked guarantees	-45
Total	951	**Total**	951

The fair value reserve first of all consists of a basic reserve for the (expected) guaranteed premiums, costs and benefits (value: 748). We do not include a market value margin, to keep the analysis a simple as possible.[9] For simplicity, we also do not consider new business but only the runoff of the existing policies. This is not to say that the effect of new business can be ignored in practice: for going-concern (ALM) analyses this aspect should definitely be taken into account. When an insurer has selected an LDI solution it is, obviously, also extremely important to update the LHP for the existing policies with additional liability hedges when new business arrives.

The insurer of this example has a significant number of insurance policies with additional profit sharing in case of high interest rates. Because this profit sharing component is essentially an interest rate option, an additional reserve is required (value: 18).[10] There is also a reserve for embedded options in unit linked policies (value: 45).[11] These options are important when the policy holders receive a guaranteed return on their investments or a guaranteed capital when the contract ends (insurers must compensate a shortfall with respect to such a guarantee). As a compensation for giving this guarantee, this insurer receives a fixed guarantee fee. The present value of the fees which are expected to be paid

[9] The idea behind the market value margin (MVM) is to assess how much a (risk-averse) investor would demand in excess of the liabilities' best-estimate value in order to be compensated for all risks that cannot be hedged. Risks which can be hedged or diversified away should have no impact on the MVM. In the current Solvency II (QIS 4) guidelines, the market value margin is approximated (marked-to-model) using a cost-of-capital approach (see CHEIOPS (2007) or CRO Forum (2008)).

[10] The profit sharing options are priced using an adaptation of Black's option formula (see Hull, 2005).

[11] The unit linked options are priced using the formula proposed by Schrager and Pelsser (2004).

in the future by the holders of these policies (value: –45) currently offsets the unit linked option reserve.[12]

The assets consist of 30% stocks (value: 285) and 70% fixed income (value: 665). These assets are annually rebalanced to the initial (30%-70%) mix. Currently, no interest rate derivatives or stock options are used by the insurer. Based on this balance sheet, the insurer's surplus is equal to 185. Because the legally required solvency level (according to the Solvency I guidelines) is equal to 46, the Solvency I ratio is equal to 398%.

2.2 Rearranging the balance sheet

We now apply the LDI concept and split the balance sheet in a return and a matching balance sheet. Figure 2.2 shows the effect of this procedure. As mentioned before, LDI is based on the idea that the liabilities are matched as good as possible with a liability-hedging portfolio (LHP). Fixed income thus naturally belongs to the LHP since these assets are used to match the interest rate sensitivity of the liabilities. The remaining assets (stocks) are part of the return portfolio (RP).

Figure 2.2: The insurer's balance sheet, but now from an LDI perspective.

The balance sheet is divided in a matching balance sheet with the liabilities and fixed income. The other assets (stocks) and the surplus are part of the return balance sheet. Because the value of the fixed income assets is smaller than the market value of the liabilities a fictive cash position is added to the matching balance sheet. This causes a short cash position (i.e., leverage) on the return balance sheet.

RETURN			
ASSETS			**LIABILITIES**
Stocks (RP)	285	**Surplus**	185
		Fictive cash (RP)	*100*
Total	285	**Total**	285

MATCHING			
ASSETS			**LIABILITIES**
Fixed income (LHP)	665	**Fair value reserve**	
		- Basic reserve	748
Fictive cash (LHP)	*100*	- Profit sharing options	18
		- Unit linked guarantee options	45
		- Present value of fees for unit linked guarantees	-45
Total	766	**Total**	766

Notice that the matching balance sheet also contains the item "fictive cash". This (fictive) cash position is needed to match the volume of the liabilities. Because (theoretically) the liabilities are exactly replicated by the corresponding assets, the matching assets should have the same volume as the liabilities. A cash position, which has (almost) no interest rate

[12] The present value of these guarantee fees may become larger (in an absolute sense) than the unit linked reserve. In this case, this present value on the balance sheet is limited to the (absolute) value of the corresponding reserve.

sensitivity, can be used to correct a volume mismatch with the liabilities without interfering with the interest rate hedges in the LHP. Note that this cash position also appears on the return balance sheet, but on the opposite side. This implies that we effectively create more leverage on the return balance sheet when the volume mismatch on the matching balance sheet increases. Due to this added leverage, the risk (and the expected return) on the return balance sheet increases.

2.3 Modeling credits

The above picture changes if the fixed income investments (partly) consist of credits. These more risky investments can, however, also be modeled within an LDI framework by decomposing them in a risk-free (Treasury bond) portfolio plus a stochastic excess term which models the additional risk and return associated with credit bonds. This stochastic excess term can be modeled conveniently in the RP with a long credits – short Treasury position. An example is given below in Figure 2.3. We here assume that 50% of all fixed income investments consists of credits.

Figure 2.3: The effect of credits on the return and matching balance sheet.

We here decompose the credit portfolio in a risk-free Treasury bond portfolio and a stochastic excess term. The fictive Treasury bonds are placed in the LHP; the stochastic excess term is modeled in the RP with a long – short position.

RETURN			
ASSETS			**LIABILITIES**
Stocks (RP)	285	**Surplus**	185
Credits (long) (RP)	333		
Fictive Treasury (short) (RP)	-333	**Fictive cash (RP)**	100
Total	285	**Total**	285

MATCHING			
ASSETS			**LIABILITIES**
Fixed income (LHP)	333	**Fair value reserve**	
		- Basic reserve	748
Fictive Treasury (LHP)	333	- Profit sharing options	18
		- Unit linked guarantee options	45
Fictive cash (LHP)	100	- Present value of fees for unit linked guarantees	-45
Total	766	**Total**	766

The balance sheet item "Fictive Treasury" in the LHP here stands for a portfolio of risk-free Treasury bonds with the same underlying maturities as the credit portfolio. The long credits – short fictive Treasury position in the RP models the stochastic excess term associated with credits.

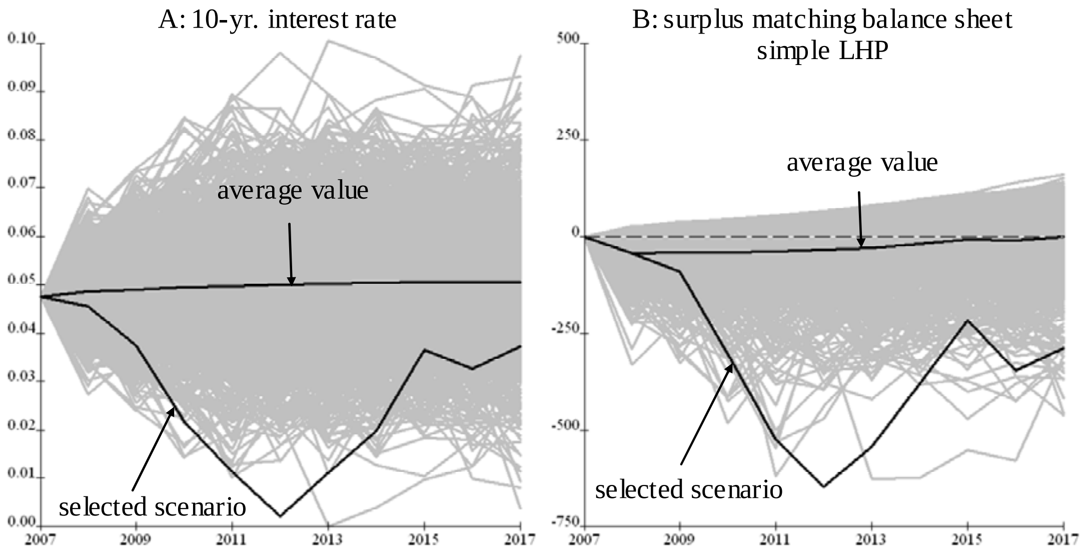
3 Construction of a liability-hedging portfolio

3.1 First analysis

We now analyze the effectiveness of the simple LHP as specified in Figure 2.2 (i.e., without credits). The value of the fixed income assets plus the fictive cash position should (theoretically) be equal to the value of the liabilities for all future scenarios. In other words, the surplus of the matching balance sheet should always be equal to zero. As a test, we generate 1,000 scenarios for interest rates and stock returns and analyze the evolution of the surplus of the matching balance sheet. Figure 3.1 shows the results of this stochastic simulation.

Figure 3.1: Evolution of the surplus of the matching balance sheet.

We here consider 1,000 different economic scenarios. Note the negative impact of a decreasing interest rate on the surplus.



Obviously, this simple LHP is not robust with respect to future economic developments. For example, for the selected economic scenario the surplus drops to almost –650 in 2012 (see Panel B). This scenario is a typical example of a low interest rate scenario (see Panel A).

To further analyze the interest rate sensitivity of this insurer we evaluate the effect of a (parallel) change of the interest rate curve at the current point of time (2007). Figure 3.2 shows how the different items of the matching balance sheet change for changes in between –4% and +4%. The item “Unit linked options” is here equal to the sum of balance sheet items “Unit linked guarantee options” and “Present value of fees for unit linked guarantees”. Figure 3.2 clearly shows that there is a large interest rate mismatch between the assets and liabilities, resulting in a strongly negative surplus for low interest rates.

Figure 3.2: The impact of parallel shifts of the interest rate curve on the matching balance sheet.

Note that the interest rate mismatch between assets and liabilities becomes large for low interest rates.

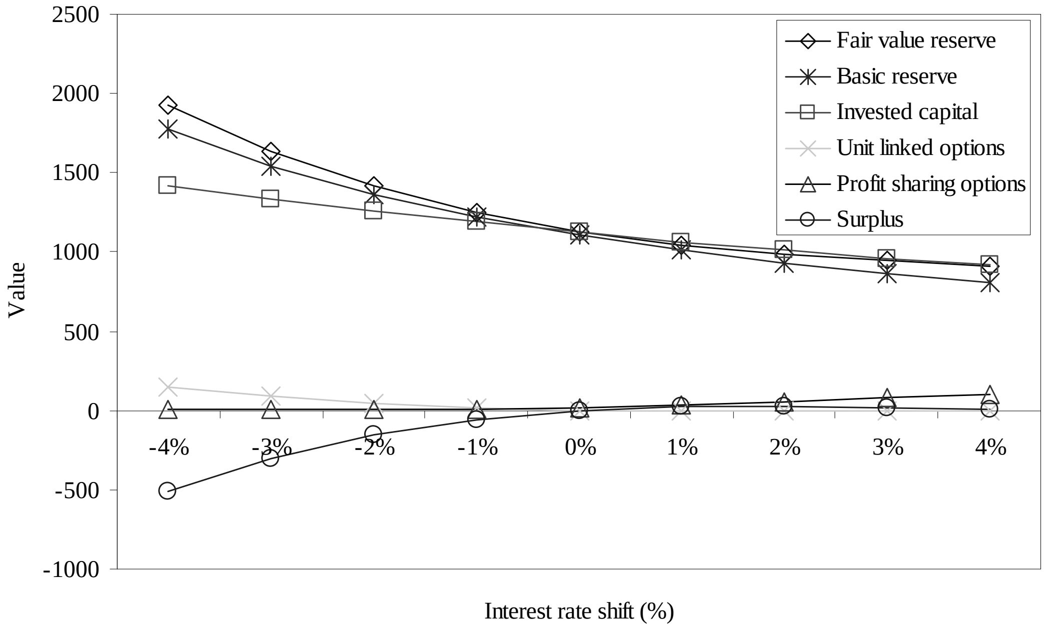
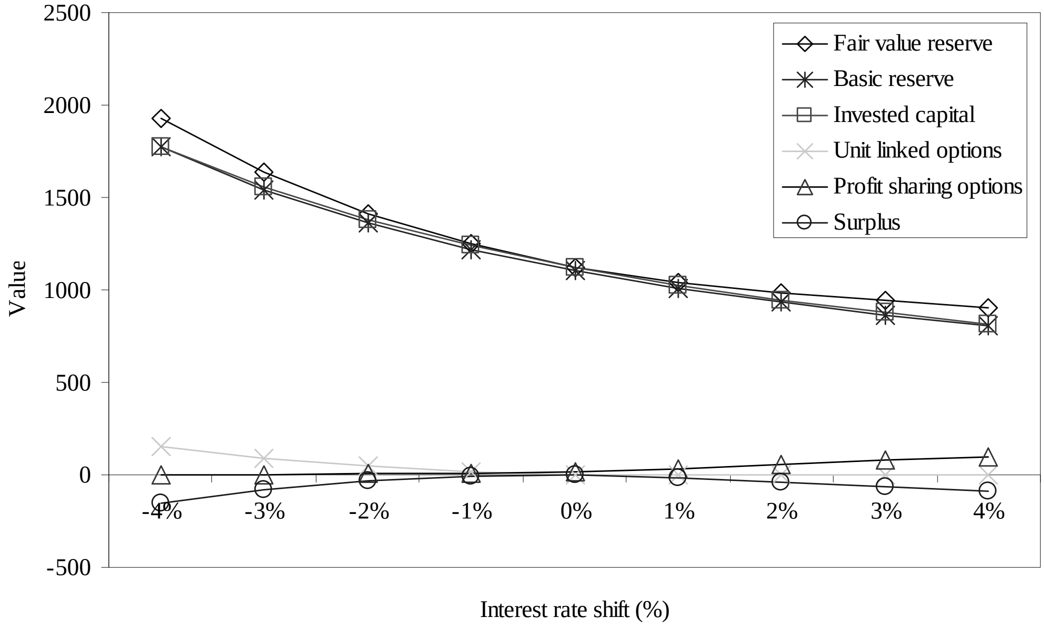


Figure 3.3: The impact of an additional swap construction on the interest rate sensitivity of the matching balance sheet.

The interest rate sensitivity of the basic reserve is now matched using the existing fixed income assets and an additional layer of swaps.



3.2 Minimizing the duration mismatch

We now add a layer of interest rate swaps to the LHP to mitigate the interest rate risk. These swaps are bought a pari (i.e., they require no initial investment) and have a range of maturities (1–10, 12, 15, 20 and 30 years) to properly match the interest sensitivity for different segments ("buckets") of the interest rate curve. We select those swaps which minimize the duration mismatch of the assets in the LHP (fixed income plus the additional swaps) with the basic reserve (the guaranteed cash flows). The effect of this swap construction is shown in Figure 3.3. The interest rate sensitivity of the basic reserve is now properly matched at the current point in time and a negative surplus is only caused by high option values for low or high interest rates.

In Figure 3.4 we study the impact of the swap construction in a stochastic simulation. In this simulation we also add an additional swap (with a maturity of 30 years) in each simulation year to further refine the initial interest rate hedge. These additional swaps are also bought a pari.

Figure 3.4: Evolution of the surplus of the matching balance sheet when an additional swap construction is used.

Notice the significant risk reduction when swaps are part of the LHP (see Panel B).

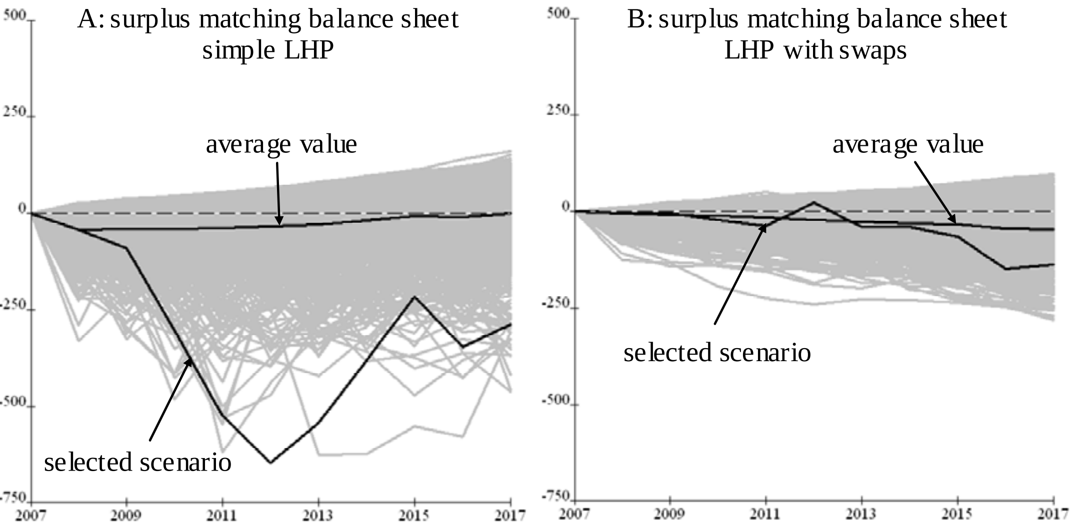
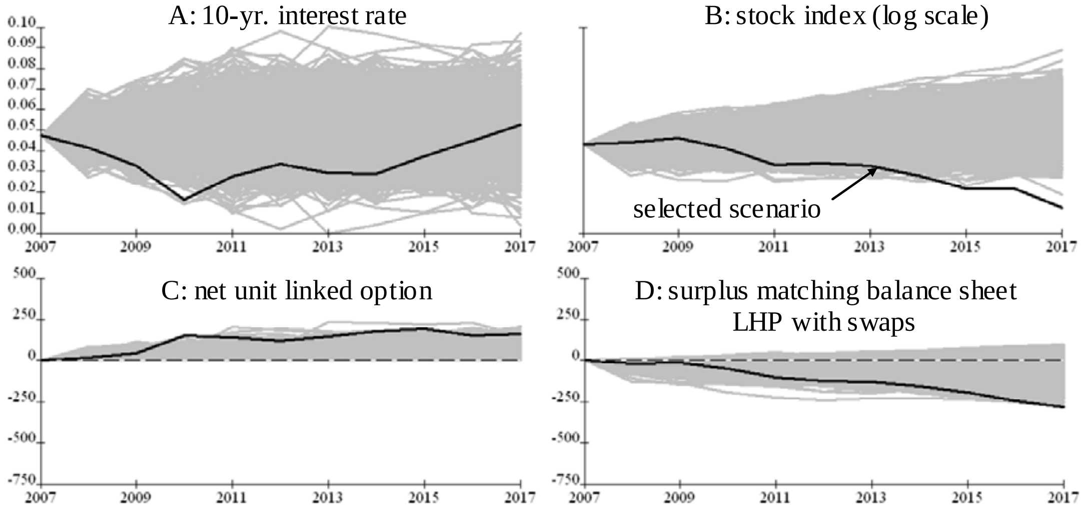


Figure 3.4 shows that the risk reduction of the swap construction is large. Note that the interest rate risk of the embedded options is also mitigated because we match the duration annually with a 30-year swap. This method does not remove the interest rate risk completely, however, since swaps are only effective for small interest rate changes.

3.3 Hedging the embedded insurance options

Because the guaranteed cash flows are now adequately matched, the remaining risks at the matching balance sheet are caused by the unit linked and profit sharing options. The most important risk at this point is the sensitivity of the (net) unit linked option to the price of the underlying stocks. This sensitivity is illustrated in Figure 3.5.

Figure 3.5: The impact of low equity returns on the surplus of the matching balance sheet. Notice the increasing value of the unit linked option (see Panel C) for a dropping stock index (see Panel B). This results in a significant decrease of the surplus of the matching balance sheet (see Panel D).



For the selected scenario the stock index performs poorly in the long run. This results in an increasing unit linked option value and a dropping surplus on the balance sheet. Averaged over all scenario's, the correlation between the stock returns and changes in the surplus is significantly positive (≈0.5), which confirms that negative stock returns are an important risk factor on the matching balance sheet.

We study the effect of (partly) hedging the option risks in the unit linked portfolio in Figure 3.6. When we hedge 50% of the unit linked option, at each point in time 50% of the option value and 50% of the option cash flows are matched; a 100% option hedge exactly matches the embedded option. We assume that these synthetic hedges are financed by selling some of the fixed income assets.

Note that Panel B, C, and D of Figure 3.6 consist of percentile plots instead of scenario plots. The 5% percentile line indicates that 5% of the 1,000 scenarios (i.e., 50 scenarios) are located below this line. The lower percentiles (e.g. for 5% and 10%) clearly shift upwards when hedging the unit linked option. The downward risks are thus significantly reduced.

Due to the nature of the unit linked guarantee option (a combined equity – interest rate option) this hedge portfolio will in practice often consists of a mixture of equity put options and (receiver) swaptions.[13] Hybrid derivatives may be a cost efficient alternative as well.

[13] In essence, we want to replicate this embedded insurance option as good as possible with financial instruments. This replicating portfolio approach has been advocated recently by different authors. For more information, see Oechslin et al. (2007) and Schrager (2008). In additional experiments (not reported here) we verified that a good hedge of the unit linked option is possible using liquid financial instruments.

Figure 3.6: The effect of hedging the unit linked option on the evolution of the surplus on the matching balance sheet.

Note that Panel B, C, and D consist of percentile plots instead of scenario plots. By comparing the lower percentiles it becomes clear that the downside risks decrease when the unit linked option is hedged.

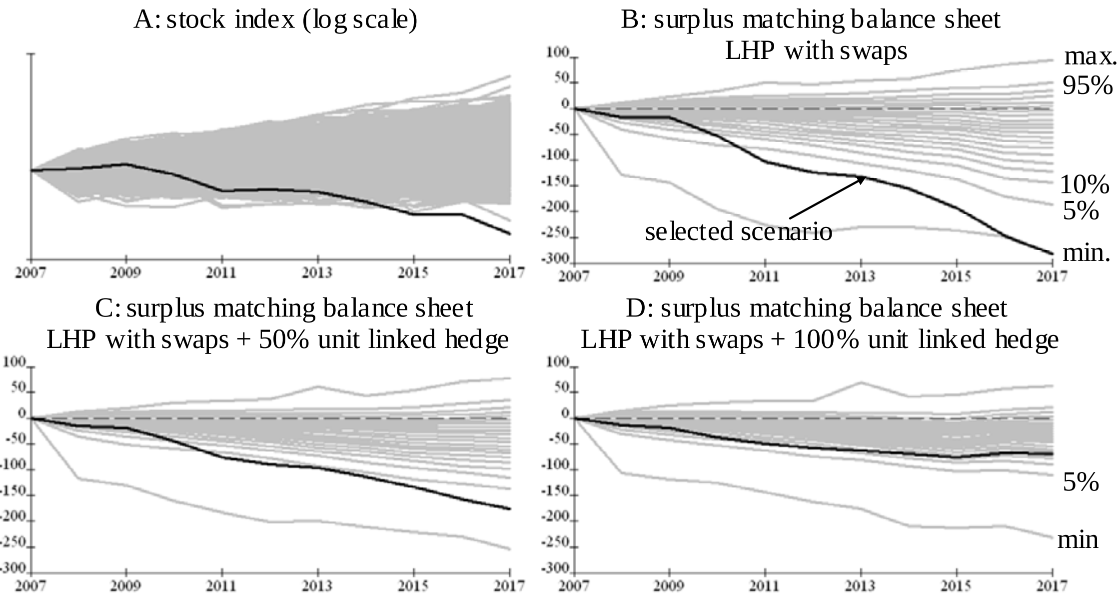
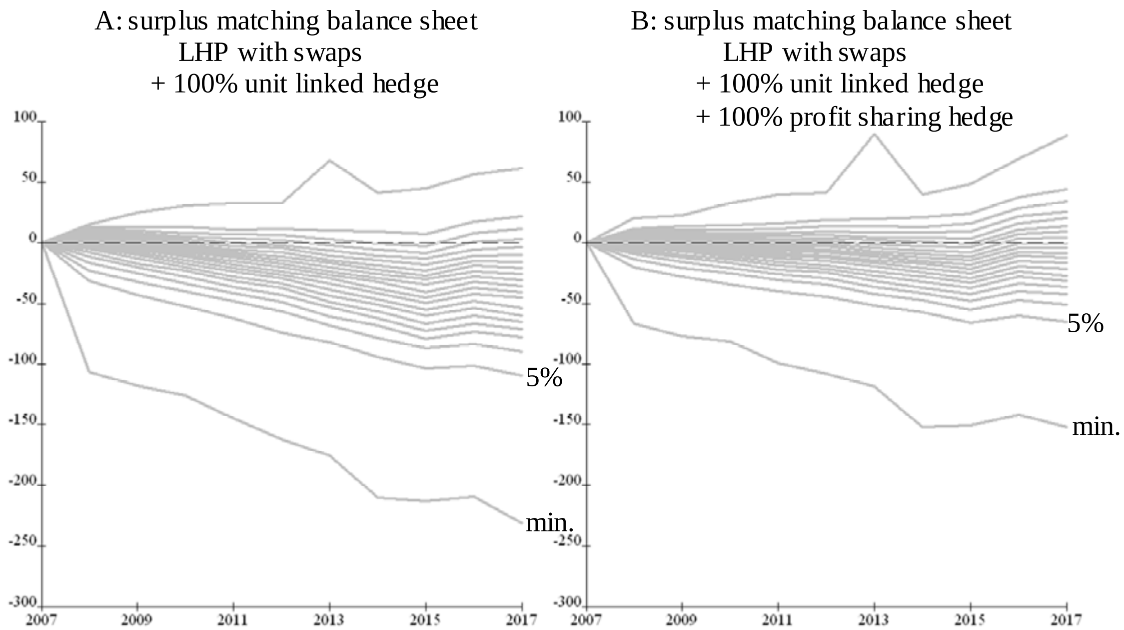


Figure 3.7: The effect of hedging the profit sharing option on the evolution of the surplus.

This percentile plot shows that the downside risks further decrease when this option is hedged.



See, for example, the recent paper by Walschots and Van Capelleveen (2008) who demonstrate that the downside risk of a pension fund can be hedged efficiently using a receiver swaption whose strike is depending on the stock market index. If the stock market drops, the strike of this swaption increases and the protection against decreasing interest rates improves. The advantage of this form of protection is that only the true risk scenarios (a combination of a low stock level and a low interest rate) are hedged. This may be cheaper than hedging the interest rate and equity risk separately.

We continue with the final risk source on the matching balance sheet: the profit sharing option. This is a pure interest rate option. Figure 3.7 shows that the risk on the matching balance sheet further reduces when we hedge these risks as well. The remaining mismatch risk is small and mainly due to the simple duration matching procedure (only once a year a new swap is bought). We have again used a synthetic option here, which exactly matches the option cash flows (and option values). Additional experiments show that in practice a good profit sharing hedge can also be constructed using a set of payer swaptions.[14]

3.4 Conclusions

We started with the simple LHP in Figure 2.2, consisting of only a fixed income portfolio in combination with a fictive cash position. We found that the mismatch of this portfolio with the actual liabilities is huge due to (i) a duration mismatch and (ii) profit sharing and guarantee options which are embedded in the liabilities. The duration mismatch can be minimized efficiently using a layer of (linear) swap contracts. The embedded options should be matched by (nonlinear) products such as payer swaptions (for the profit sharing option) or a mixture of receiver swaptions and equity put options (or hybrid options) for the unit linked guarantees. The resulting, optimal LHP portfolio is shown in Figure 3.8.

It is important to note that with a relatively modest investment (63) the downside risks on the matching balance sheet can be reduced significantly. This is illustrated in Figure 3.9, where we compare the quality of the initial LHP portfolio with fixed income in Figure 2.2 with the optimized LHP in Figure 3.8.

Interestingly, the added option hedges do not appear to have a negative impact on the average surplus on the matching balance sheet (compare the average values in Panel A and B). In fact, the average surplus increases slightly because the strongly negative scenarios are almost entirely eliminated (the upside scenarios are also suppressed, but this effect is less pronounced). Setting up a proper LHP portfolio thus appears to be very attractive: the downside risks are suppressed significantly while the average return of the LHP (plus the fictive cash position) remains sufficiently high to match the expected return of the liabilities.

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[14] Payer swaptions can be used to hedge the profit sharing option because (ignoring the unit linked option) we can decompose the liabilities as: guaranteed cash flows + profit sharing = guaranteed cash flows + max(interest rate – threshold, 0) x notional = “assets A” + payer swaptions. Another (equivalent) decomposition would be: {guaranteed cash flows + (interest rate – threshold) x notional} + max(threshold – interest rate, 0) x notional = “assets B” + receiver swaptions.

Figure 3.8: LDI balance sheet with the optimized LHP portfolio.

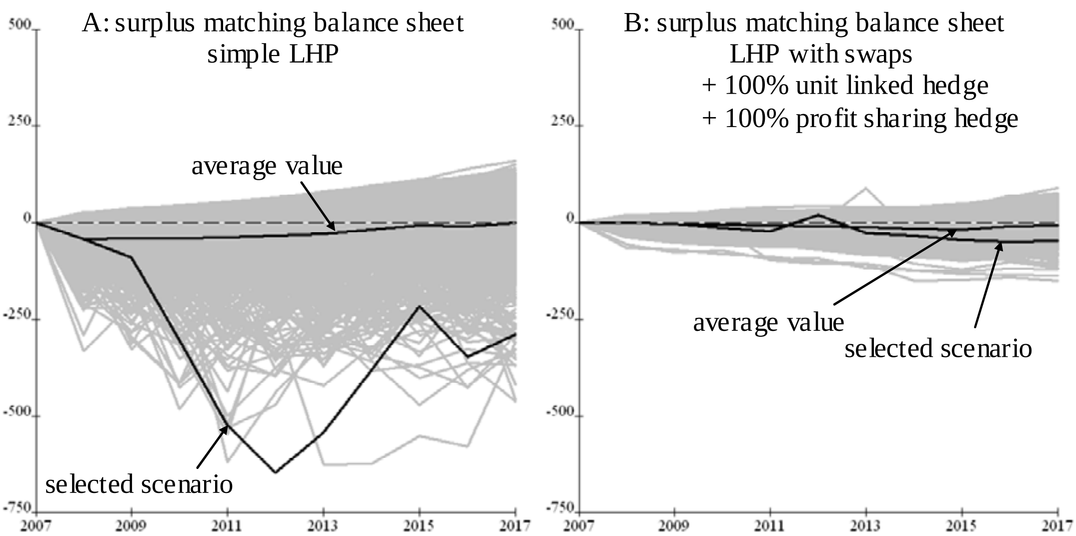
A layer of swaps is bought a pari (i.e., without an initial investment) to match the duration of the LHP with the duration of the basic reserve. In addition, a 30-year swap is bought in each future year to update this initial hedge (these instruments are not visible on the initial balance sheet). The embedded (profit sharing and unit linked) options are hedged using synthetic options which mimic the behavior of the embedded options in the liabilities.

RETURN			
ASSETS			**LIABILITIES**
Stocks (RP)	285	**Surplus**	185
		Fictive cash (RP)	100
Total	285	**Total**	285

MATCHING			
ASSETS			**LIABILITIES**
Fixed income (LHP)	602	**Fair value reserve**	
		- Basic reserve	748
Fictive cash (LHP)	100	- Profit sharing options	18
		- Unit linked guarantee options	45
Swaps (LHP)	0	- Present value of fees for unit linked guarantees	-45
Hedges embedded options (LHP)			
- Profit sharing options	18		
- Unit linked guarantee options	45		
Total	766	**Total**	766

Figure 3.9: A comparison of the quality of the simple LHP and the optimized LHP.

Note that the downward risks on the matching balance sheet are reduced significantly by adding swaps and hedges for the unit linked and profit sharing options in the liabilities.



4 Analysis complete balance sheet

We now proceed with an analysis of the complete balance sheet. For simplicity, we consider the stylized situation in which the insurer does not pay taxes or dividends. The results are shown in Figure 4.1. Panel A shows the evolution of the surplus in case of the original balance sheet in Figure 2.1. Results in Panel B (for the return–matching balance sheet in Figure 2.2) are quite similar because the underlying assets and liabilities are the same. Small differences occur, however, because (annual) rebalancing has a different effect when we consider a single balance sheet with annual rebalancing (as in Panel A) or two different balance sheets (as in Panel B). Panel C clearly shows the effect of the optimized LHP in Figure 3.8: the downside risks are reduced significantly.

To further reduce the risks, we can reduce the percentage of stocks and invest more money in fixed income. This has almost no impact on the risks on the matching balance sheet because the interest rate risks are minimized by the swap construction in the LHP.[15] The impact on the RP balance sheet is significant, however, because the amount of leverage decreases (as well as the allocation to stocks). This effect is illustrated in Figure 4.2, where we show the evolution of the surplus on the matching and total balance sheet.

In Panel A and C we consider the situation in Figure 3.8, where 87% of the volume of the liabilities is matched with fixed income. In Panel B and D we consider the situation where 100% of the volume of the liabilities is matched (i.e., fixed income has a value of 766 and stocks a value of 185). Comparing Panel C with Panel D shows that the downside (stock) risk decreases when we reduce the amount of leverage (and the exposure to stocks). This, however, has a negative impact on the return: the average surplus after 10 years is lower in Panel D than in Panel C. The average return on the surplus also decreases: from 10.3% to 8.8%.

It is possible to further reduce the downward stock risk using an appropriate (put or collar) stock option strategy. It is, of course, also useful to construct a more diversified return portfolio using (Markowitz) optimization techniques.[16] Both approaches are well established and are therefore not explored further in this paper.

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[15] The initial swap portfolio is (slightly) adjusted because more fixed income is available on the matching balance sheet in this case.

[16] See for example Steenkamp (2005).

Figure 4.1: Evolution of the surplus for 1,000 different economic scenarios.

We here consider the original balance sheet (see Figure 2.1), the simple LDI balance sheet in Figure 2.2 and the optimized LDI balance sheet in Figure 3.8. Note that the downward risks decrease significantly when we optimize the LHP (see Panel B and C).

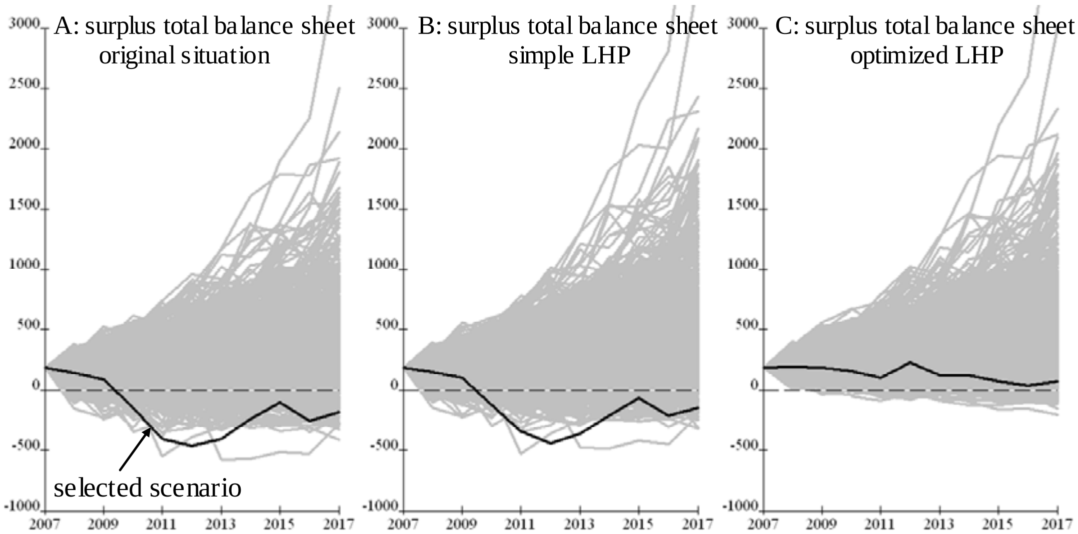
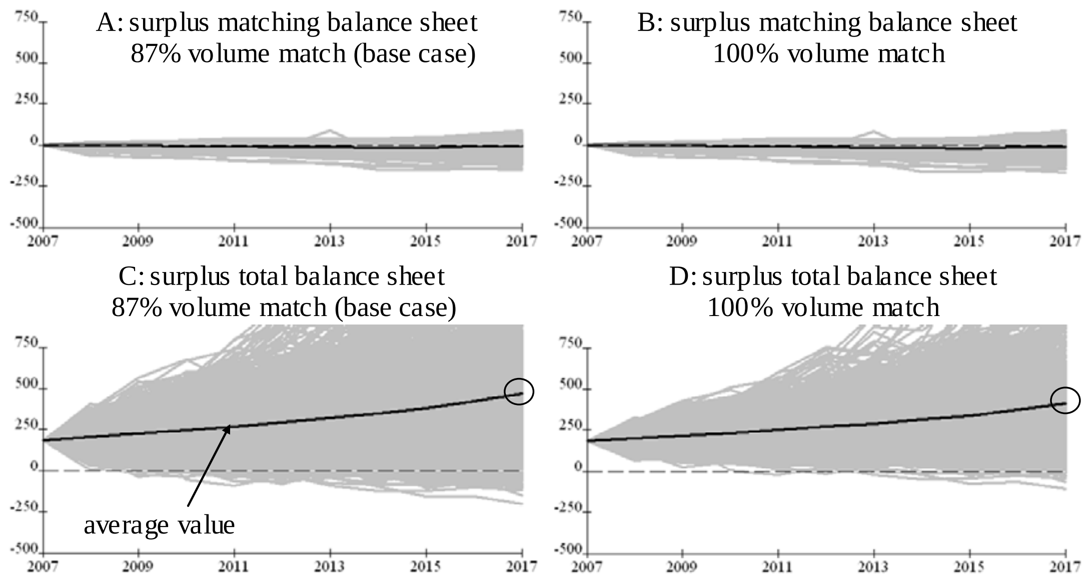


Figure 4.2: Impact of the volume of the liabilities that is matched by the LHP.

In Panel B and D the volume of the liabilities is completely matched with fixed income. This has almost no effect on the mismatch risk on the matching balance sheet (compare Panel A with Panel B). The effect on the total balance sheet is significant, however, because leverage on the RP balance sheet is eliminated in Panel B and D and the allocation to stocks decreases. This reduces both the downside risk and the expected return.



5 Conclusions

The liability-driven investing (LDI) concept can be used to divide the asset allocation problem for a life insurer into separate, more tractable problems. This improves the (complicated) asset allocation decision. The basic idea behind LDI is to split the balance sheet into two separate balance sheets: one for the liabilities and the matching assets and one for the other (return) assets and the surplus.

Using a realistic example, we show how an insurer can match the liabilities, including all embedded options, using a liability-hedging portfolio (LHP) in combination with a (fictive) cash position. The fictive cash position also emerges as a short cash position (i.e., leverage) on the return balance sheet. We demonstrate that setting up a proper LHP portfolio is very attractive because the liability-driven risks are suppressed significantly while the average return of the LHP (plus the fictive cash position) remains sufficiently high to match the expected return of the liabilities.

The final step is an optimization of the assets on the return balance sheets. Since the liability-driven risks are already suppressed by the matching assets, these assets can be optimized using well-known (Markowitz) optimization techniques or (equity) hedge strategies. The LDI approach thus stimulates insurers to address all risks embedded in the insurance liabilities and facilitates the subsequent optimization of the return assets.

References

1. Amenc, N., L. Martellini, P. Foulquier and S. Sender (2006), *"The Impact of IFRS and Solvency II on Asset-Liability Management and Asset Management in Insurance Companies"*, EDHEC. Available at www.edhec-risk.com.

2. Amenc, N., L. Martellini and V. Ziemann (2007), *"Asset-Liability Management Decisions in Private Banking"*, EDHEC. Available at www.edhec-risk.com.

3. Boender, G (2007), "Investors Must Approach LDI with Caution", *Global Pensions*, March 2007, 24.

4. CHEIOPS (2007), "QIS 4 – Technical Specifications", CEIOPS-DOC-23/07, December 2007. Available at http://ec.europa.eu/internal_market/insurance/docs/solvency/qis4/technical_specifications_en.pdf.

5. CRO Forum (2008), "*Market Value of Liabilities for Insurance Firms*", July 2008. Available at http://www.croforum.org/publications.ecp.

6. Hull, J.C. (2005), *"Options, Futures, & Other Derivatives"* (6th Edition), Prentice Hall.

7. Maton, B. (2007), "A Term with Many Different Meanings", *Financial Times*, April 20.

8. Mindlin, D. (2006), "The Myth of the Liability Benchmark", *Global Pensions*, October 2006, 16-17.

9. Oechslin, J., Aubry, O., Aellig, M., Käppeli, A., Brönnimann, D. and A. Tandonnet (2007), "Replicating embedded options in life insurance policies", *Life & Pensions Magazine*, February 2007.

10. PIMCO (2007), "Bond Basics: Liability-Driven Investing", May 2007. Available at www.pimco.com.

11. Schrager, D.F. and A.A.J. Pelsser (2004), "Pricing Rate of Return Guarantees in Regular Premium Unit Linked Insurance", *Insurance: Mathematics and Economics*, 35 (2), 369-398.

12. Schrager, D. (2008), "Replicating Portfolios for Insurance Liabilities", *Aenorm*, no. 59, April 2008, 57-61.

13. Steenkamp, T (2005), *"Asset Allocation and Portfolio Construction"*, Riskmatrix, Driebergen, The Netherlands.

14. Towers Perrin (2006), "*Solvency II Introductory Guide*", CEA Insurers of Europe.

15. Walschots, B. and H. van Capelleveen (2008), "Hybride Bescherming", *VBA Journaal*, no. 2, Summer 2008, 40-47 (in Dutch).

16. Zenios, S.A. and W.T. Ziemba (2006, editors), "*Handbook of Asset and Liability Management*", Volume 2(1) in Series of Handbooks in Finance, Elsevier Science, 2006.

17. Zenios, S.A. and W.T. Ziemba (2007, editors), "*Handbook of Asset and Liability Management*", Volume 2(2) in Series of Handbooks in Finance, Elsevier Science, 2007.

18. Ziemba, W.T. and J.M. Mulvey (1998, editors), "*Worldwide Asset and Liability Management*", Cambridge University Press, 1998.