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Towards a new scenario generator with consistent short and long term economic scenarios

Part II: A frequency domain Vector AutoRegressive scenario framework

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Testing existing models

Why test VAR models anyway?

- *Do VAR models and, more important, the way they are applied in both academic and practical ALM, indeed lead to scenarios that are consistent with the empirical knowledge obtained from the first question…*

- Estimating a VAR model does not automatically lead to scenarios that are consistent with the empirical behavior of the time series variables used for estimation
- Reasons
  - Imprecise estimates in case of small samples
  - Data representation
  - Model order
  - Estimation procedure

Methodology

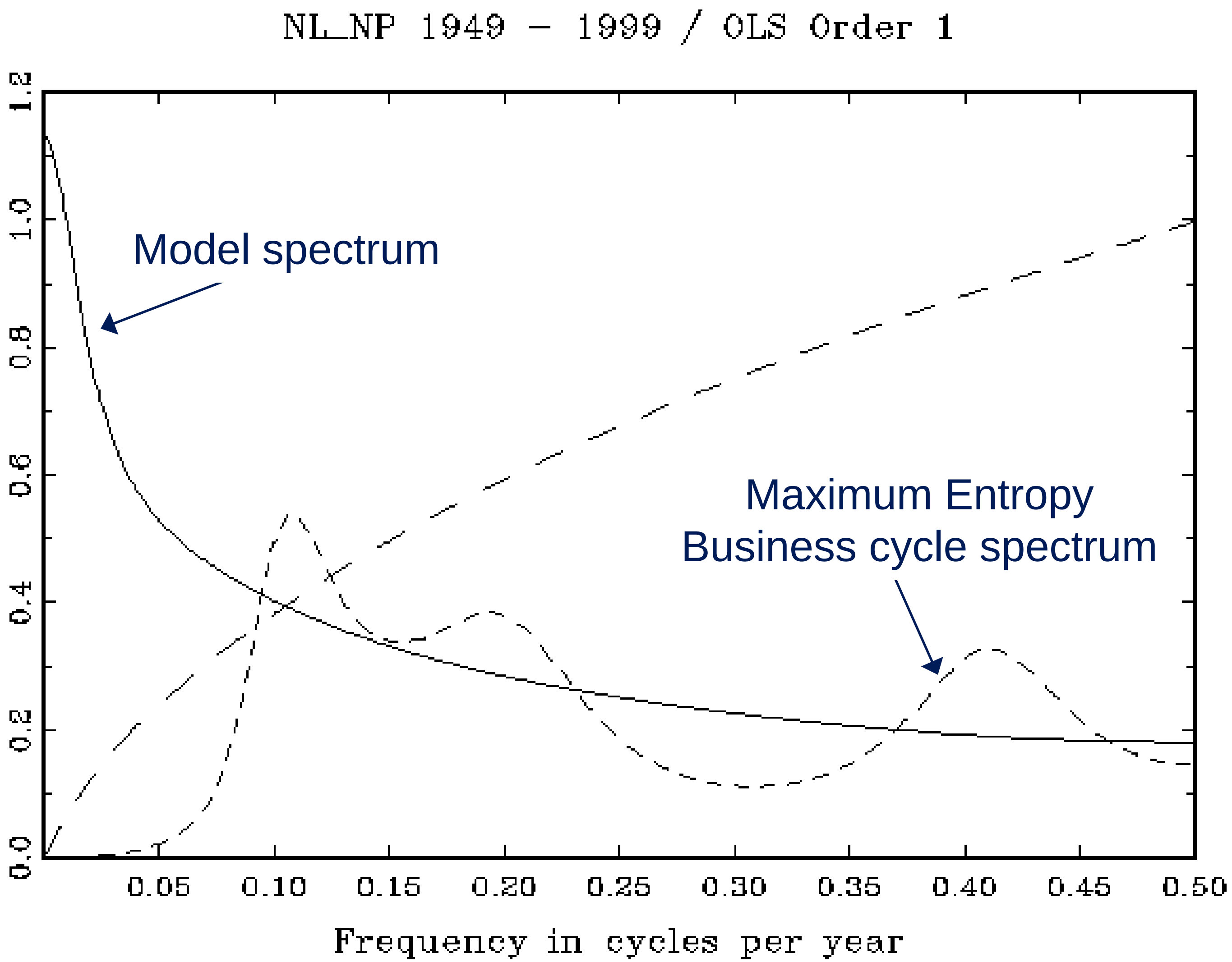
- Specific (ALM) scenario applications of VAR models used to construct typical (literature and practice) combinations of
  - 1. Data representation
  - 2. Model order
  - 3. Estimation procedure
- Reasons
  - Consistency with stylized facts (same data)
  - All model information available
  - Flexibility (for example impact of changing model order)
  - Relevant information on consistency issue

Data representation

- Six dimensional VAR models
- Dutch annual data for the postwar 1949-1999 period
- Two representations
  - 1. Conventional in terms of growth rates
  - 2. Filtered business cycle components of growth rates

	Representation 1	*Representation 2*
National Product Index	$\Delta\log(NP)$	$[1/15, 0.5]\ \Delta\log(NP)$
Consumer Price Index	$\Delta\log(PI)$	$[1/15, 0.5]\ \Delta\log(PI)$
Real Industry Wage Index	$\Delta\log(WI)$	$[1/15, 0.5]\ \Delta\log(WI)$
Short Term Interest Rates	SR	$[1/15, 0.5]$ SR
Long Term Interest Rates	LR	$[1/15, 0.5]$ LR
Real Equity TRR Index	$\Delta\log(TR)$	$[1/15, 0.5]\ \Delta\log(TR)$

Comparing spectral densities



- Normalized on the same total variance

Summary of results - I

- Current applications of VAR models in many respects do not produce results that are consistent with the empirical knowledge.
- Often model order 1 is used by defintion which can be too restrictive. Also consider higher model orders.
- One is often not well aware of the dynamic model properties. Consider the spectral densities of the model.
- "Long term model properties based on only one long term sample". Consider using different samples and different observation frequencies for different frequency ranges.
- Complex relations such as the level effect in interest rates cannot be modelled well.
- Time or phase shifts are often modelled correctly (robust).
- Univariate dynamics are difficult to model well in multivariate models. High order models perform best here.

Summary of results - II

- Using conventional representations of the data, conventional VAR models are often not capable of describing both the low and high frequency fluctuations well at the same time. Conventional models put the focus on the frequencies that are by chance the most important in the data representation chosen for the estimation of the model. By first filtering time series and then estimating models on those filtered times series, better estimates can be obtained for the relevant frequency range.
- Estimating a low order VAR model that, because of its dimension, in principle has sufficient flexibility to describe the expected univariate dynamics is not enough to guarantee that the model will actually also describe these dynamics. In this case, unconstrained multivariate models have the tendency to "average” across the univariate dynamics.

Summary of results - III

- High order models for high dimensional processes may offer too much flexibility, thereby producing too rich dynamics that blur the “true” underlying dynamics.
- This causes a difficult problem of conflicting model requirements. On the one hand, a low order model with in principle enough flexibility is not enough to produce the right model dynamics while on the other hand, a high order model performs better but offers too much flexibility in the dynamics of the model.
- A possible way out of the preceding problem is to apply appropriate restrictions on a model in order to have the best of both model orders. However, conventional ways of restricting the models can actually actually worsen instead of improve the results. Apparently the models are restricted in the wrong way.

A frequency domain Vector AutoRegressive scenario framework

General

- ... *and, if this is not the case, what <u>modifications</u> can be made to resolve the shortcomings of the current applications?*

- New framework should encompass conventional applications

- Components
  - Multifactor VAR (MVAR) models
  - Truncated VAR models
  - EigenValue Restricted VAR (EVR VAR) models
  - Adjusting dynamics of a VAR model by hand
  - Testing and calibrating instruments

Multifactor VAR (MVAR) models

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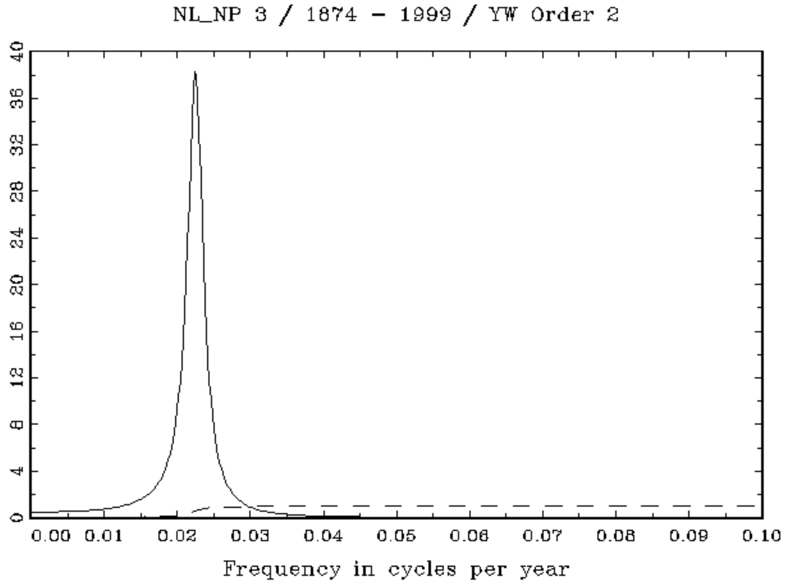
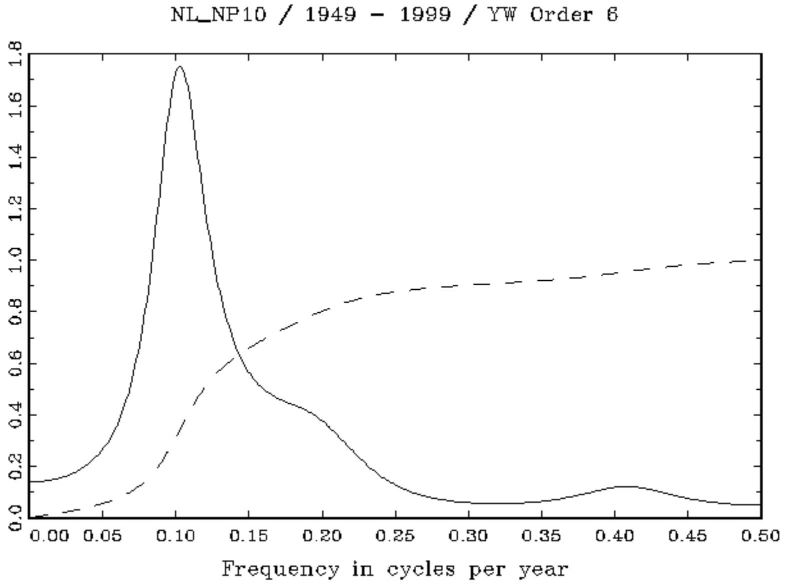
- Separate VAR models for non-overlapping frequency ranges of time series obtained from the zero phase frequency filter
- Separate and therefore flexible, transparent and high quality modeling of the behavior of its variables in different frequency regions
- Different samples and observation frequencies for different components
- Explicit modeling of stochastic trends, the behavior of the variables at the ultra low frequencies
- Modeling of complex dependencies between the stochastic properties of the model in different frequency ranges (factor dependencies)
- Better awareness of dynamic model properties

Example:
Log Dutch National Product Index

- Trend model
  - Deterministic log linear trend with average growth of 2.7%
- Long wave model
  - 5 year observations of 1870-1999 long wave component
  - AR(2) model
- Business cycle model
  - 1 year observations of 1949-1999 business cycle component
  - AR(6) model





Confidence intervals

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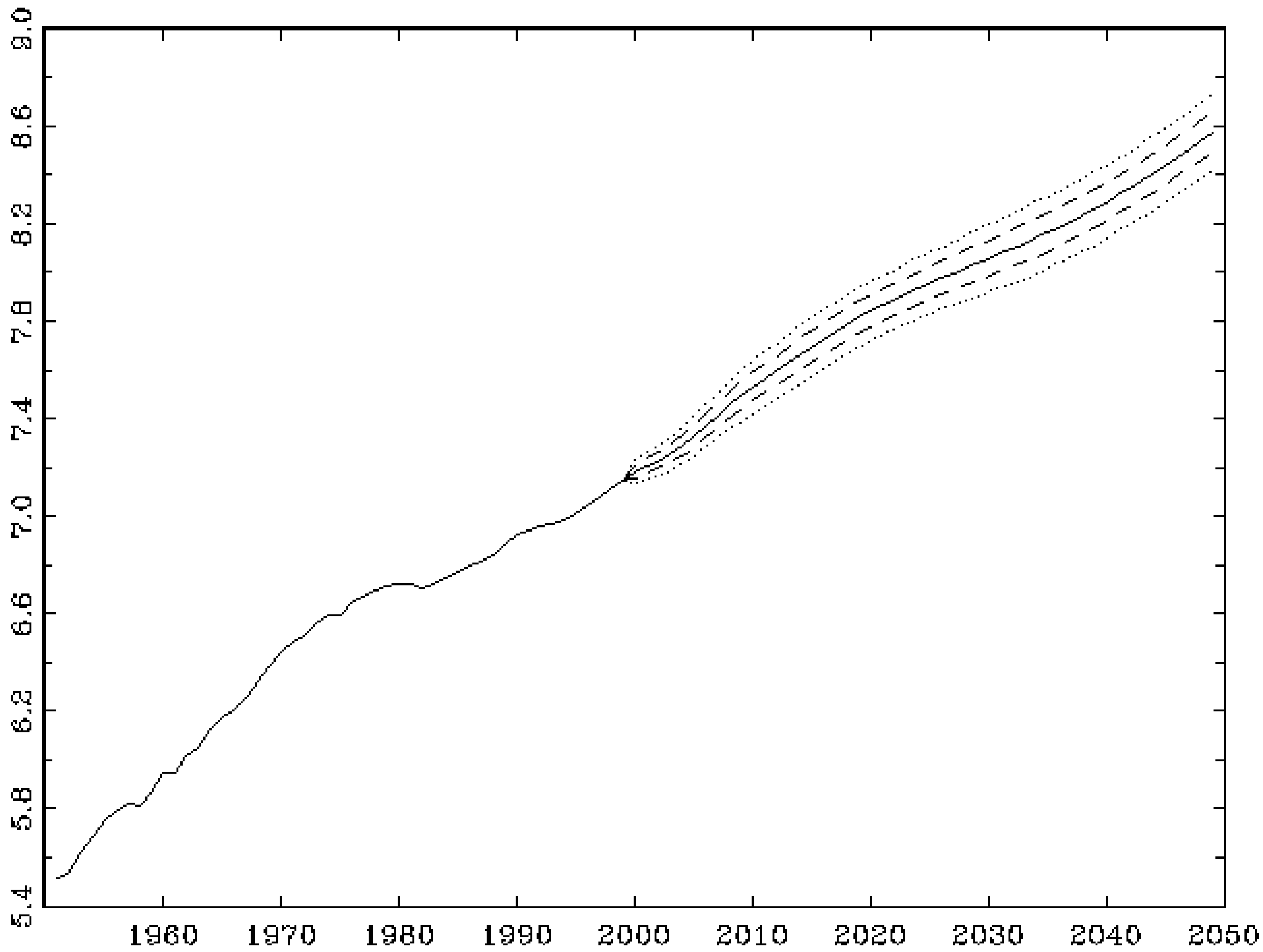


- 0.1%, 5%, 50%, 95% and 99.9% percentiles

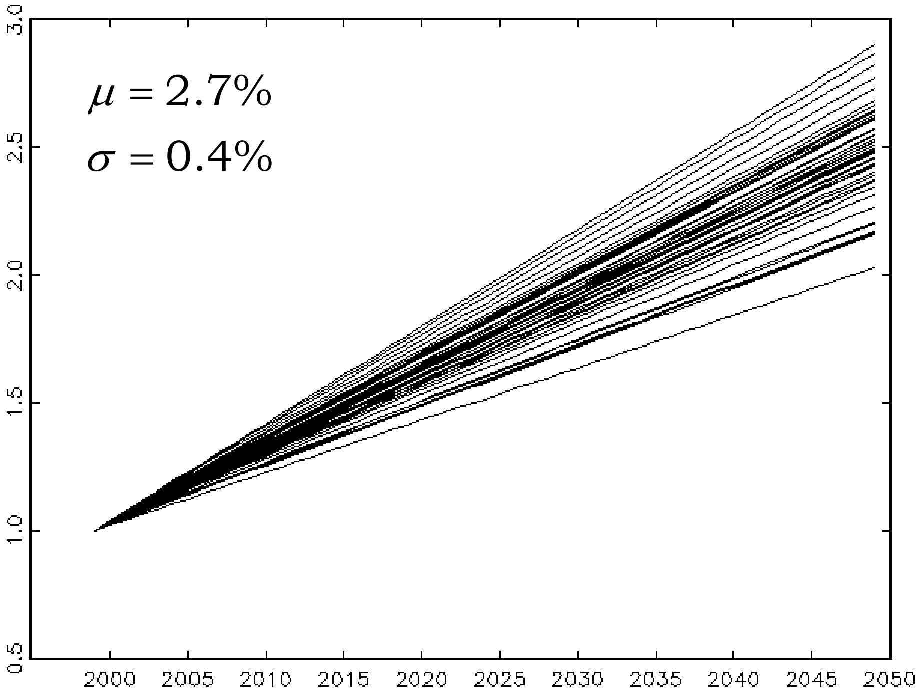
Stochastic trend - I

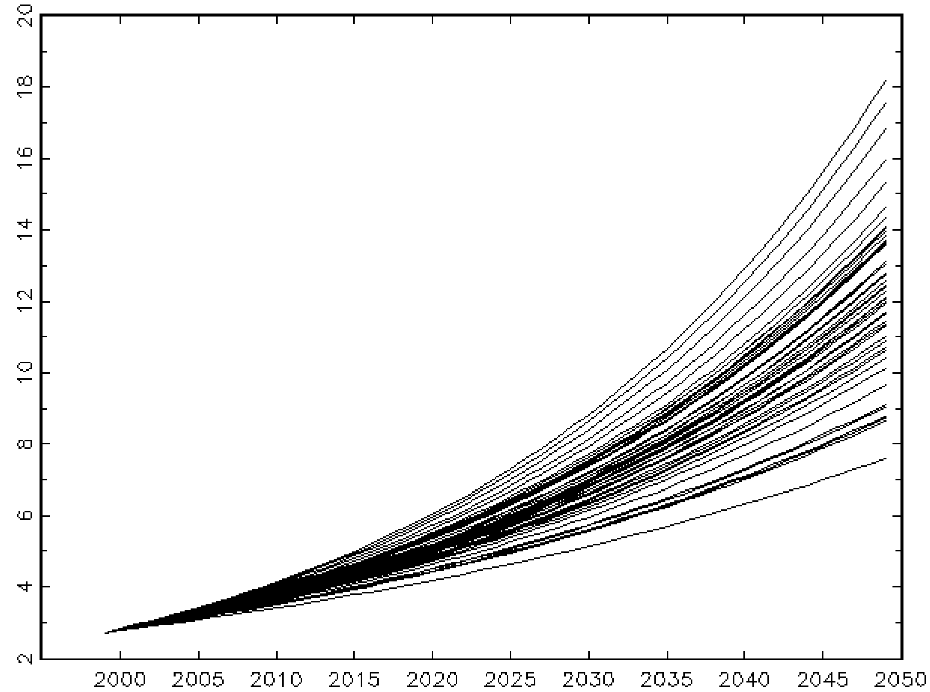
- Conventional stochastic trend model

$$x_t - x_{t-1} = \varepsilon_t$$
$$\varepsilon_i \sim N(0, \sigma^2)$$
$$V(x_{t+h}) = \sum_{i=1}^{h} \sigma^2 = h\sigma^2$$

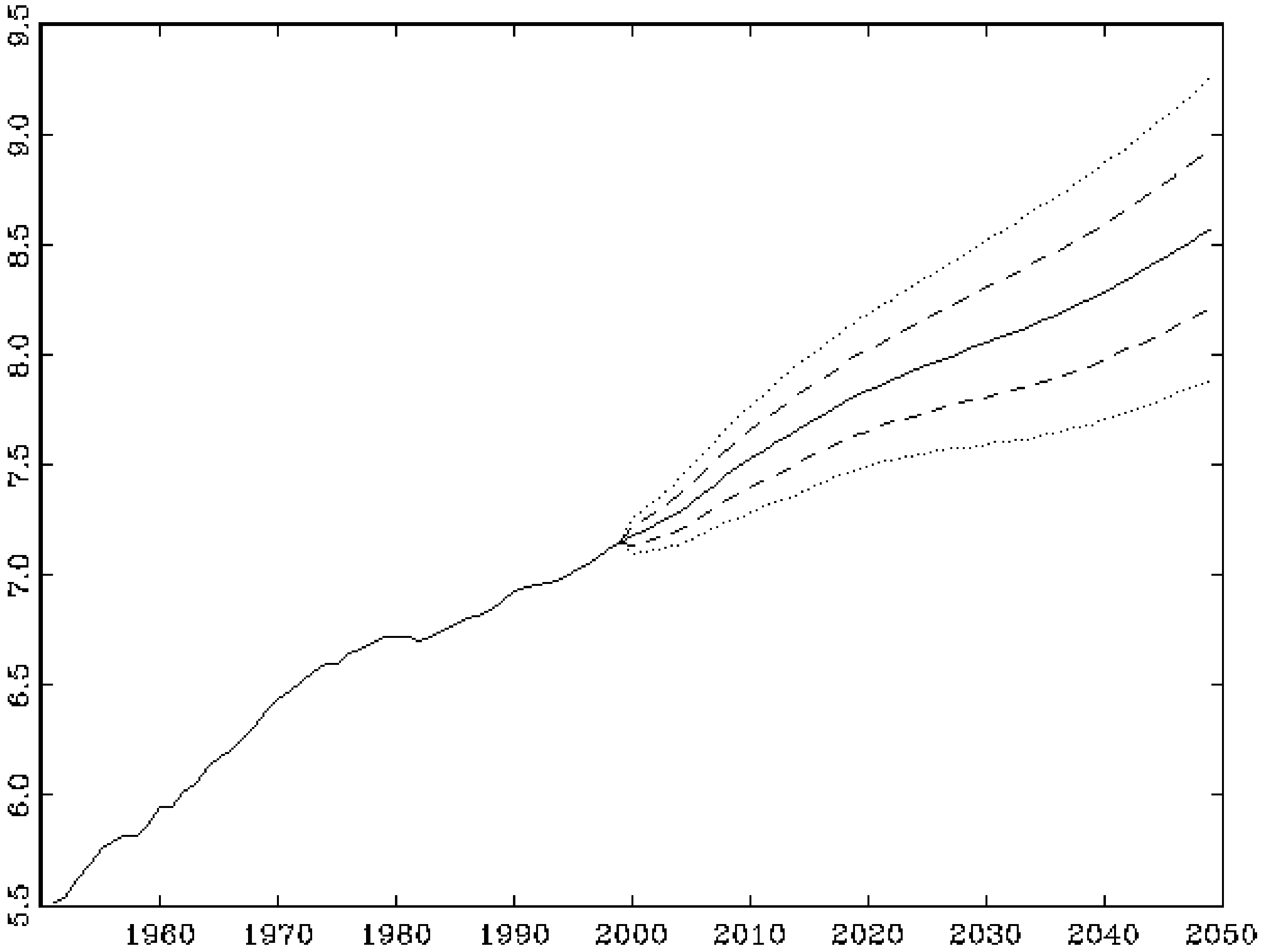
- Log-Linear stochastic trend in MVAR model

$$x_{i,t+h} = x_t + \varepsilon_i \cdot h$$
$$\varepsilon_i \sim N(\mu, \sigma^2)$$
$$V(x_{i,th}) = h^2 V(\varepsilon_i) = h^2\sigma^2$$





Stochastic trend - II



- Explicit stochastic trend modeling
- Other simple trend models available for non-trending variables

Volatility of stochastic trends

- Overall conditional volatilities that are consistent with the uncertainty in historical time series
- Cross sectional data of long term averages and trend values for the relevant variables from other, comparable, countries
- Long term averages from individual sample
  - Longer than longest periodlength modelled in other components
  - Correct for autocorrelation / unsmooth
- Survey results and expert opinion
- Unconditional volatility that is consistent with historical volatility

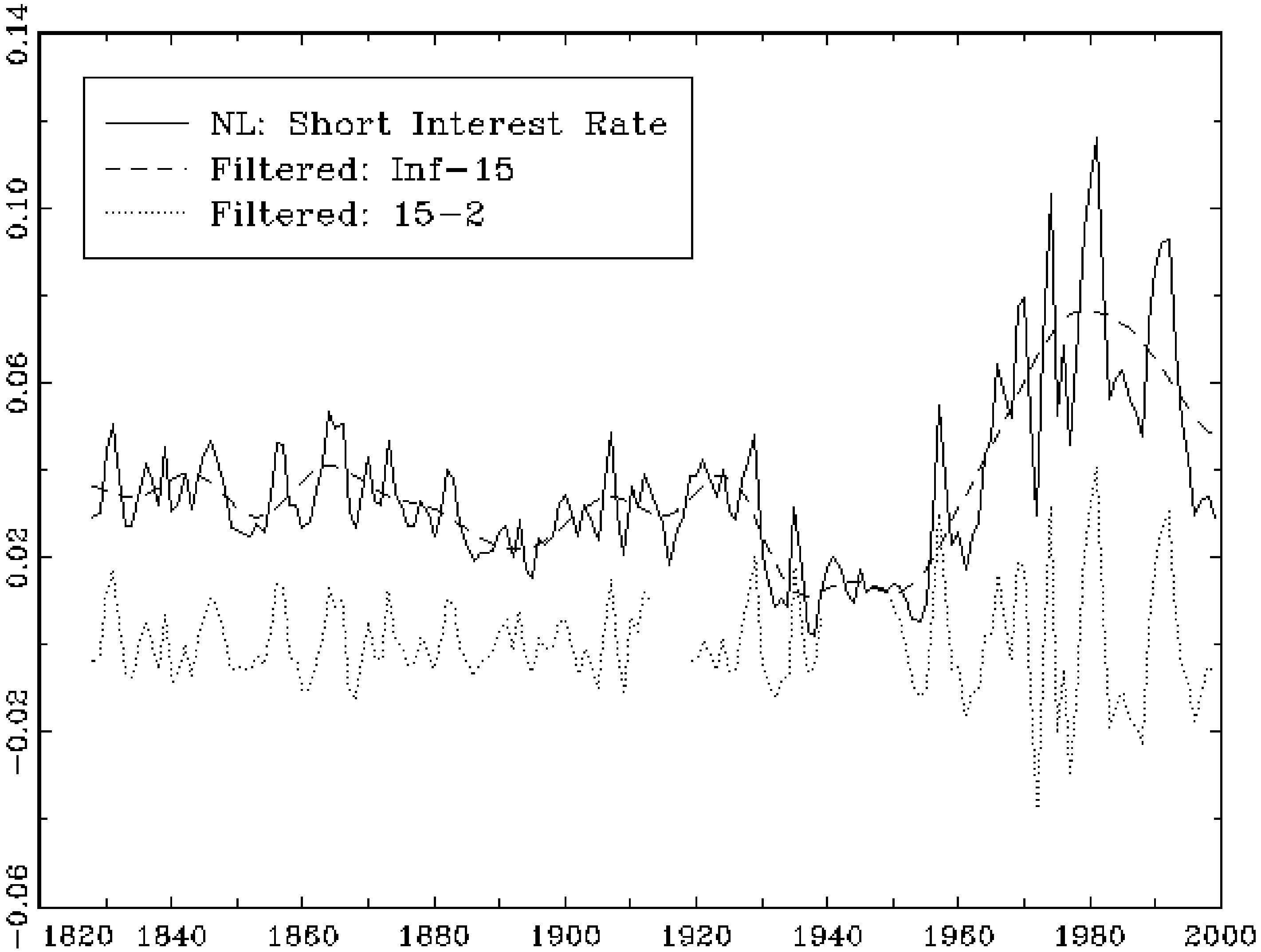
Factor dependencies: Level effect

- Level effect in interest rates describes a positive relation between the volatility of the business cycle fluctuations and the underlying level of the interest rates

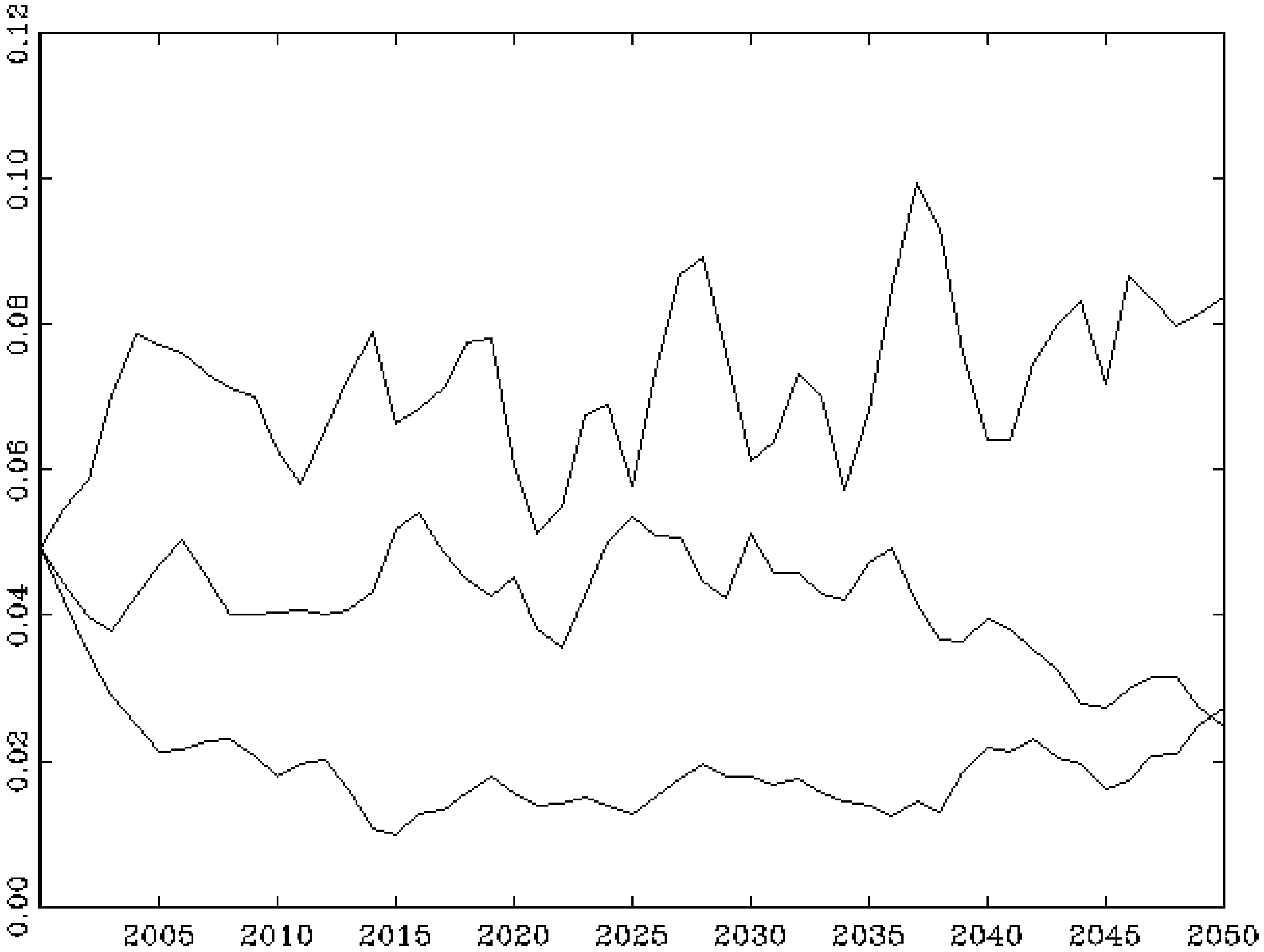
- Can be explicitely modelled in MVAR models by directly linking the properties of the business cycle model to the simulated state of the underlying level

- Can equally well be applied to more complex types of factor dependencies, for example
  - State dependent business cycle dynamics
  - State dependent asset correlations (Copulas)

Level effect - Historical



Level effect - Scenarios

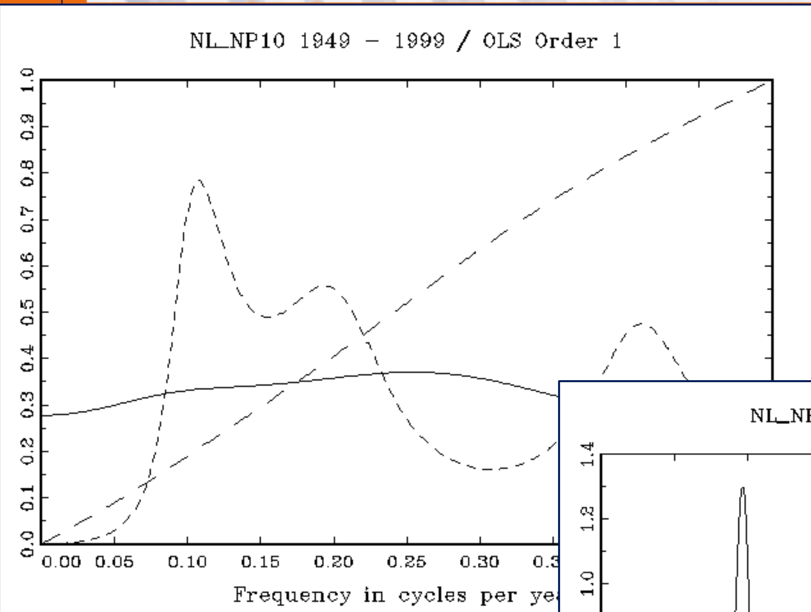


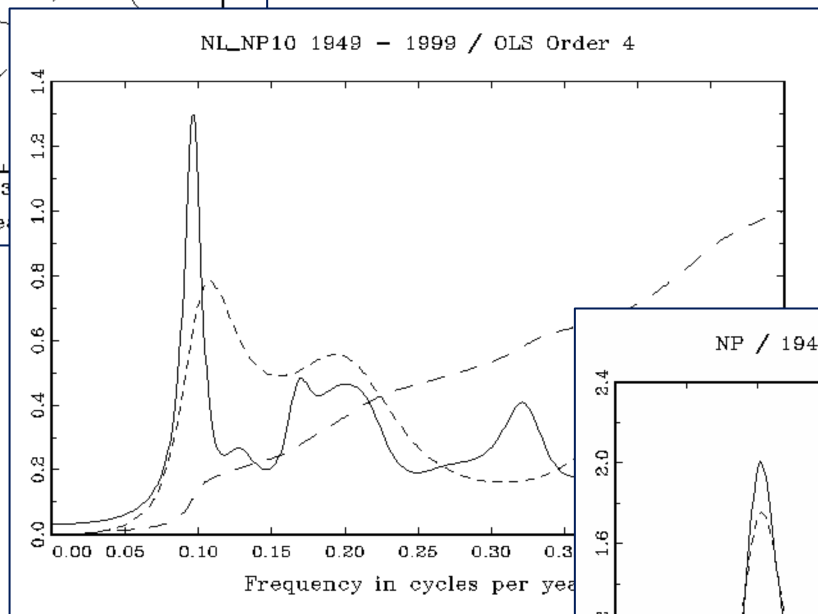
EigenValue Restricted (EVR) VAR models

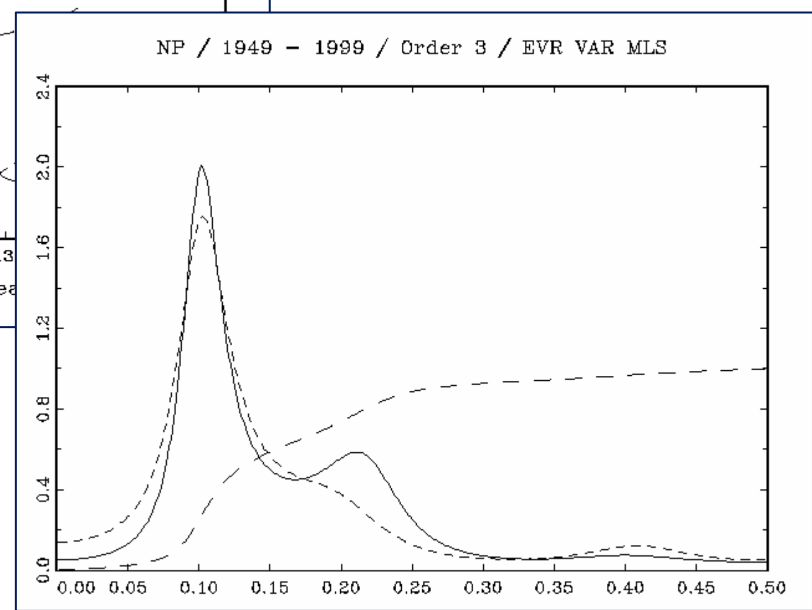
- The eigenvalues and eigenvectors of the parameter matrices of a VAR model directly determine its dynamic behavior
- Idea is to fix the eigenvalues of a VAR model at some pre-specified values and estimate the optimal corresponding eigenvectors
- Can be used to either
  - Enforce a certain type of dynamics on a low order VAR model or
  - Reduce the dynamics of a high order VAR model.
- Hereby restrictions can be put on the (eigenvalue and eigenvector) parameters of a model that directly determine its dynamic behavior instead of on the conventional parameters
- Also offers the possibility to adjust the model dynamics by hand

Example: Business cycle National Product

- Six dimensional business cycle VAR model
- Unrestricted VAR(1): "Averaging" dynamics
- Unrestricted VAR(4): Too much flexibility
- Solution: EVR VAR(3) model







Conclusions Part II

- A frequency domain framework consisting of
  - the zero phase frequency filter
  - spectral analysis
  - and various special classes of VAR models

  constitutes a clear, flexible and consistent approach for generating and analyzing scenarios of economic variables in a way that meets the current and future requirements put on economic scenarios

- Directions for future research
  - Implementation of new scenario framework in ORTEC models
  - Analysis of higher frequency properties (monthly, weekly, daily)
  - Consequences of new scenarios for strategic (ALM) policy decisions
  - Integration with various levels of decision making