VAR Models For Generating Scenarios In ALM: Do's And Don'ts

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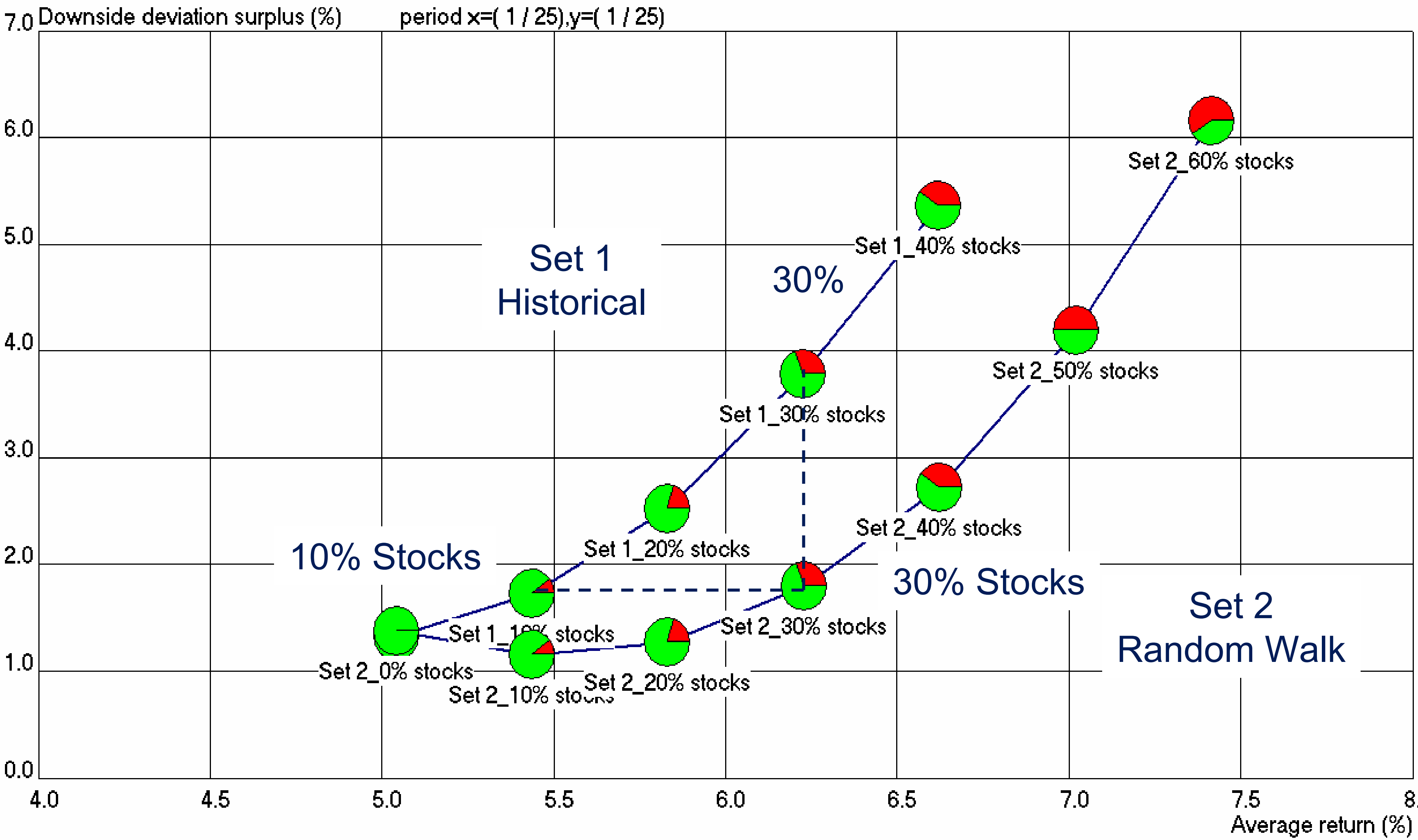
Dr. Hens Steehouwer[1,2]

ORTEC bv
P.O. Box 4074
3006 AB Rotterdam
Tel. 010 498 6660
Fax. 010 498 6667
info@ortec.nl

1. ORTEC Centre for Financial Research
2. Affiliated with the Econometric Institute of the Erasmus University Rotterdam

Relevance and Objective



- 10% vs 30% stocks due to "slightly" different specification of scenario generator!
- Understand VAR models together with practical scenario pitfalls and remedies

Contents

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What's a VAR model?

- VAR stands for Vector Auto Regressive
- Time series models in which a vector of variables (x_t) is described as a linear function (v and A_i) of historical values of the same variables (x_{t-i}) plus a multivariate Normally distributed error process (ε_t)

$$x_t = v + A_1 x_{t-1} + A_2 x_{t-2} + \ldots + A_p x_{t-p} + \varepsilon_t$$

$$x_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{n,t} \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \quad A_i = \begin{bmatrix} a_{i,1,1} & a_{i,1,2} & \cdots & a_{i,1,n} \\ a_{i,2,1} & a_{i,2,2} & \cdots & a_{i,2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{i,n,1} & a_{i,n,2} & \cdots & a_{i,n,n} \end{bmatrix} \quad \text{for } \; i = 1, \ldots, p$$

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{n,t} \end{bmatrix}, \quad E(\varepsilon_t) = 0, \quad E(\varepsilon_t \varepsilon_s') = \begin{cases} \Sigma & \text{for } \; s = t \\ 0 & \text{for } \; s \neq t \end{cases}$$

- First (V)AR models from Yule (1927) and Slutzky (1927): Sunspot periodicity
- In macroeconomics since Sims (1980): Alternative for large structural models

What are they used for?

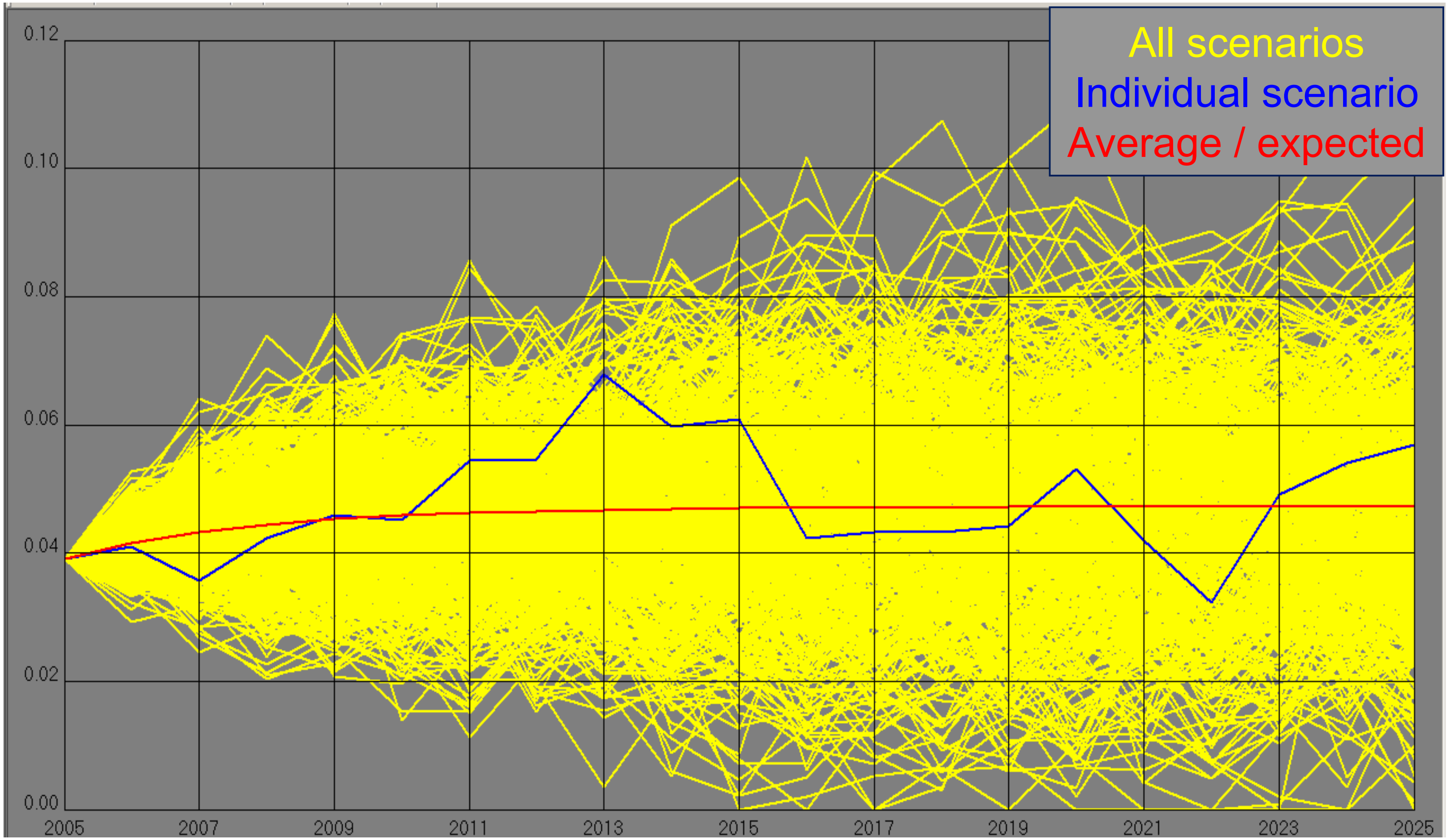
- Predictions of future path of economic variables

- Understanding dynamics of (macro) economic systems

- Generating scenarios / modeling uncertainty of economic variables
  - Asset and Liability Management (ALM)
  - Asset allocation problems
  - Yield curves, investment returns, inflations rates, exchange rates, etc.

- Numerous practical and scientific applications, for example
  - Boender et al. (2005) - ALM
  - Campbell and Viceira (2002) - strategic portfolio selection
  - Dert (1995) - multistage stochastic programming
  - Damm (1995) and Steehouwer (1998) - yield curve modeling
  - Kim et al. (1999) - RiskMetrics LongRun

Scenarios from a VAR model



Why are they used?

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- Simple linear format
- Reduced form of many (unknown) structural models
- Can describe almost any covariance stationary time series process
  - Expected values
  - Variance and (contemporaneous) correlations
  - Auto- and cross correlations (dynamics)

What is their biggest problem?

- Imprecise estimates
  - Pro of great flexibility is at the same time a con
  - Especially when estimation needs to take place on limited economic data
  - Lack of prior model structure leads to poor quality and uncertain estimates.
  - Runkle (1987) “*VAR models may let the data speak for themselves but they are not speaking very loudly.*”

- Subjective choices
  - Data representation (levels, delta’s, log’s, filtered series, etc.)
  - Model order (*p*) and selection criteria (AIC, SC, HQ, CAT, etc.)
  - Estimation method (OLS, SUR, Yule-Walker, Bayesian, restricted, etc.)

- All can have an enormous impact on
  - Statistical properties of VAR models and scenarios
  - Outcomes of ALM, investment or economic models that use VAR models
  - <u>Important policy decisions that are based on these model outcomes</u>

Example

- Pension Fund
- Two scenario sets for five variables
  - Annual data 1970-2002
  - Horizon 25 years
  - VAR(1) model estimated by Yule-Walker method
- Set 1. Historical (auto-) covariance structure
  - Historical averages are overruled
- Set 2. Identical but now with a Random Walk for stocks

Assumptions

Set 1: Historical

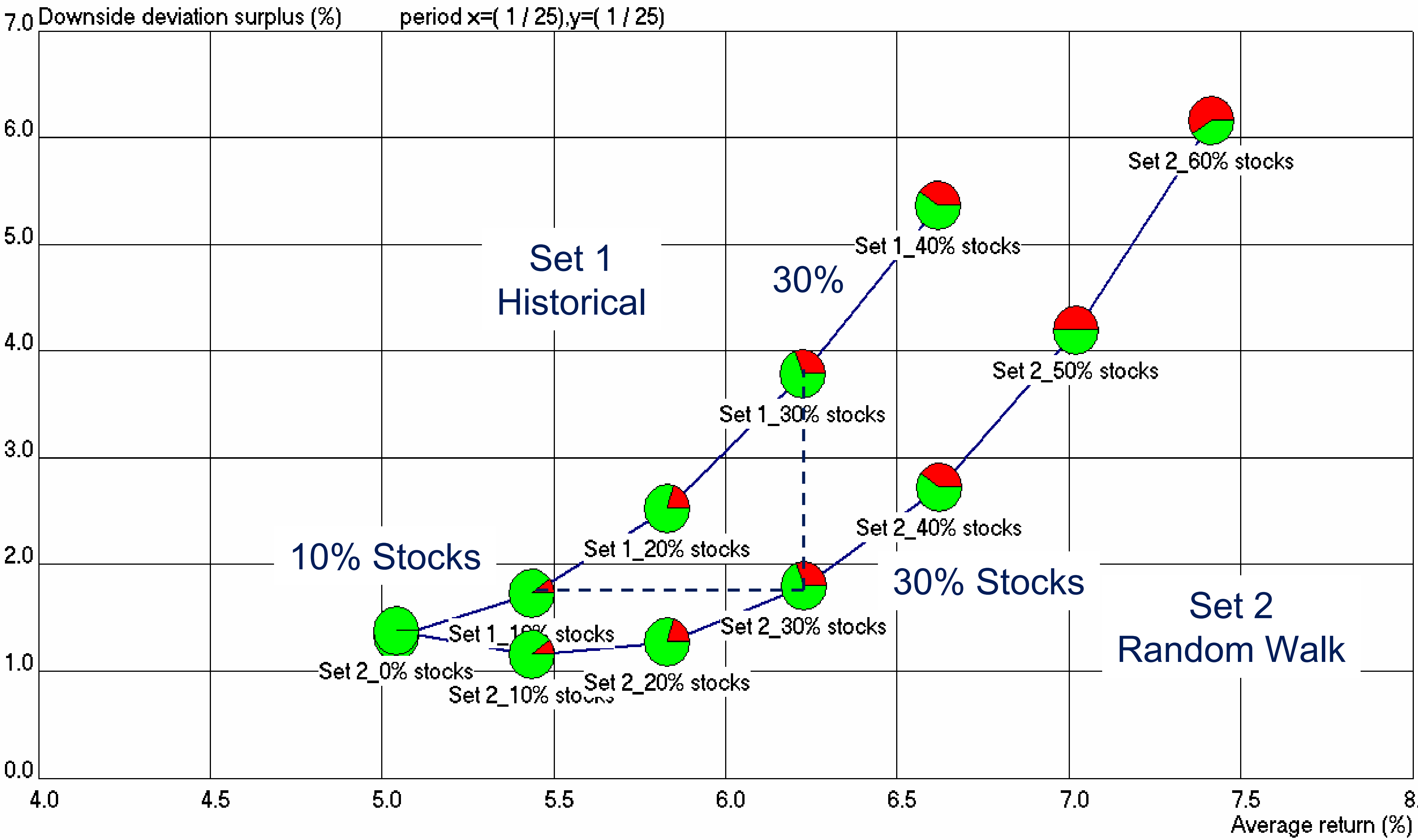
	Avg	Stdev	Correlations (t,t)					Cross correlations (t,t-1)				
Price Inflation	2.25%	2.80%	1.0					0.9	0.9	0.4	0.5	-0.3
Wage Inflation	3.25%	3.90%	0.9	1.0				0.9	0.9	0.3	0.4	-0.3
Short Interest Rate	4.25%	2.90%	0.4	0.3	1.0			0.2	0.3	0.6	0.5	0.0
Long Interest Rate	5.25%	1.60%	0.6	0.4	0.8	1.0		0.5	0.4	0.7	0.8	-0.1
Stocks Returns	9.00%	22.10%	-0.2	-0.2	-0.2	-0.1	1.0	-0.1	-0.1	-0.1	0.1	0.0

Set 2: Random Walk

	Avg	Stdev	Correlations (t,t)					Cross correlations (t,t-1)				
Price Inflation	2.25%	2.80%	1.0					0.9	0.9	0.4	0.5	0.0
Wage Inflation	3.25%	3.90%	0.9	1.0				0.9	0.9	0.3	0.4	0.0
Short Interest Rate	4.25%	2.90%	0.4	0.3	1.0			0.2	0.3	0.6	0.5	0.0
Long Interest Rate	5.25%	1.60%	0.6	0.4	0.8	1.0		0.5	0.4	0.7	0.8	0.0
Stocks Returns	9.00%	22.10%	-0.2	-0.2	-0.2	-0.1	1.0	0.0	0.0	0.0	0.0	0.0

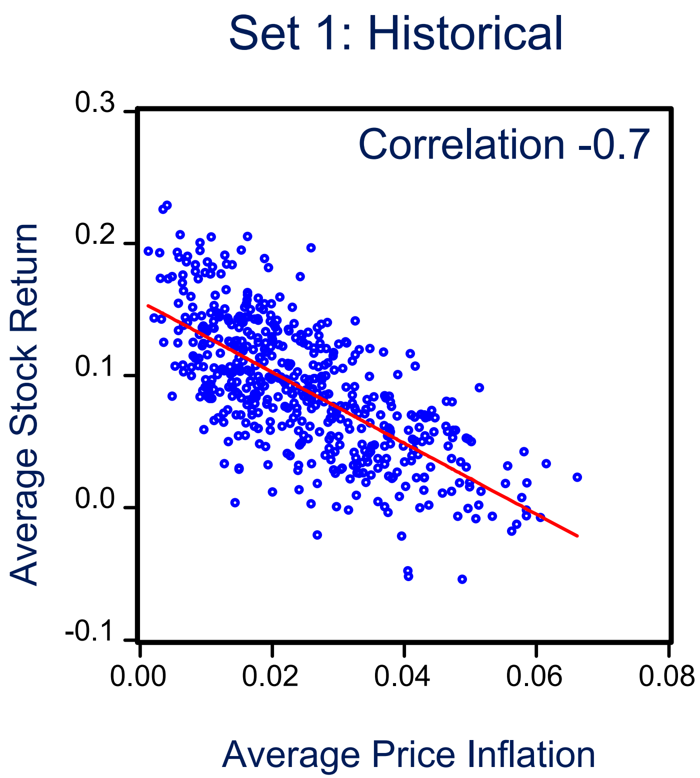
- Here Random Walk as all 1st order auto- and cross correlations 0

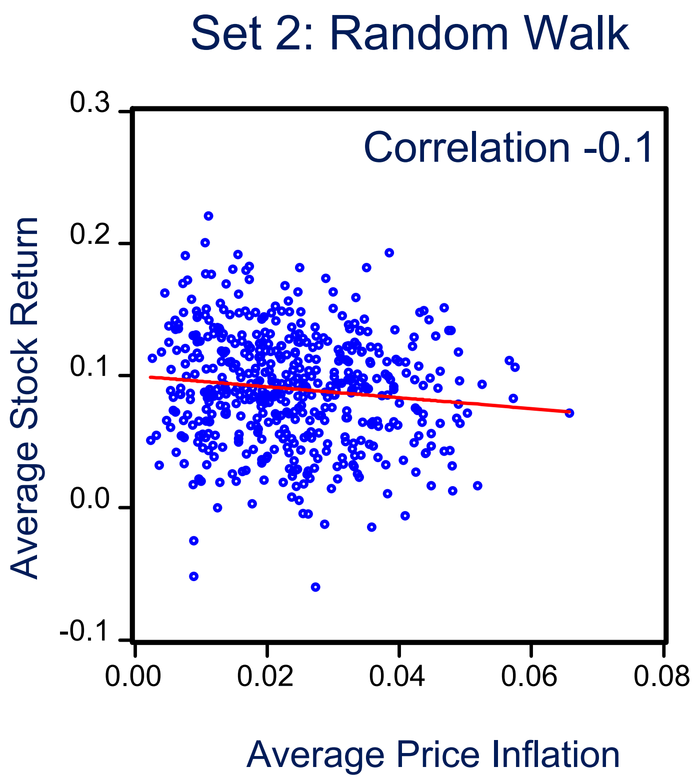
Results



- 10% versus 30% in stocks due to small differences in correlations

Explanation





- 25 year averages per scenario
- Unchanged short term stock – inflation correlation (-0.2)
- Improved (less harmful) long term stock – inflation correlation

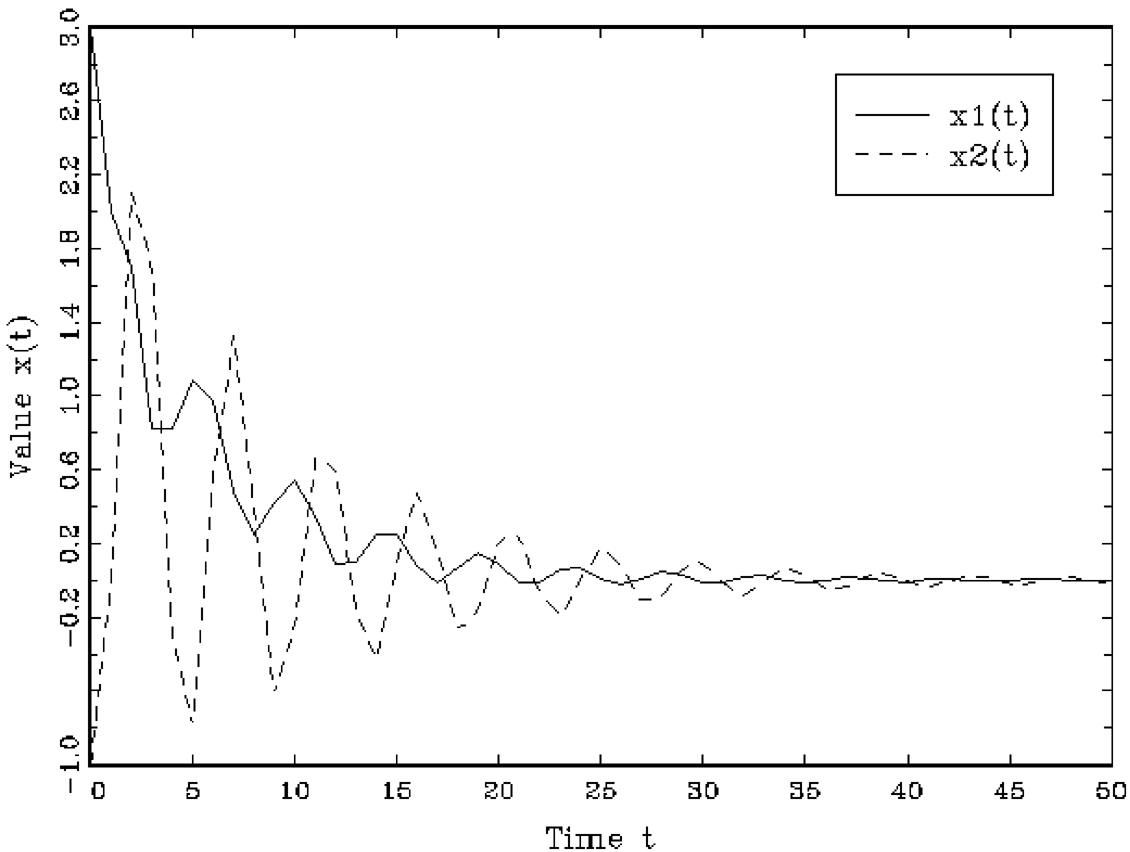
How to understand VAR dynamics?

- Not directly from the conventional parameter matrices
- 1. Expected values and (conditional) auto covariance structure
  - Can be calculated from so called Yule-Walker equations.
- 2. Impulse response functions
  - Give a shock to every variable and see the effect on the other variables.
- 3. Deterministic simulation
  - Simple dynamic linear system without stochastic error terms
- 4. Eigenvalues and eigenvectors of parameter matrices
- 5. Multivariate spectral densities
  - Example in Appendix

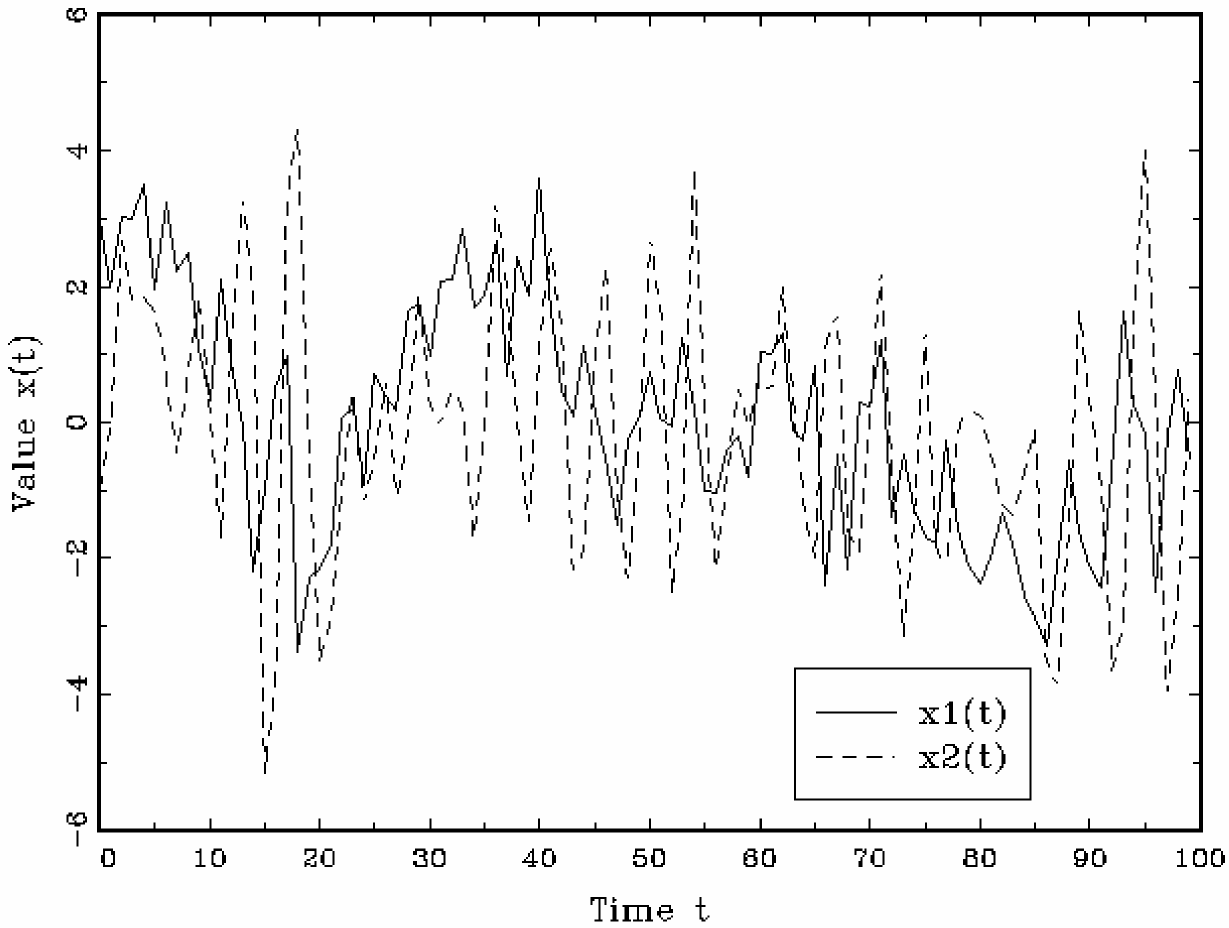
Deterministic simulation

- 2 dimensional VAR(2) model

$$\begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.3 \\ 0.4 & 0.3 \end{bmatrix} \begin{bmatrix} x_{1,t-1} \\ x_{2,t-1} \end{bmatrix} + \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & -0.7 \end{bmatrix} \begin{bmatrix} x_{1,t-2} \\ x_{2,t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \quad \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$



Stochastic simulation



- x_1: Slowly moving pattern together with fluctuations with a length of 4 to 5 years
- x_2: Mostly fluctuations with a length of 4 to 5 years
- Short term fluctuations highly correlated
- x_1 leads x_2 by 1 to 2 years for the short term fluctuations

Do's and Don'ts - I

- Know *what* to model before *how* to model.
  - Empirical knowledge should precede (advanced) models and theory.
  - Clearly formulate the assumptions underlying your scenarios.

- Incorporate forward looking information in your scenarios.
  - The past need not tell us everything about the future.

- Check if your scenarios are indeed consistent with your assumptions about their (stochastic) behavior.

  ➔ *Example 1*

- Understand the (multi-period) dynamic behavior of your model.
  - This goes much further than just a covariance matrix of returns.
  - Starting point for understanding ALM or asset allocation results

Example 1: Check consistency

- Generate scenarios
  - of 14 economic variables with an horizon of 15 years
  - in accordance with (2nd moment) statistics of historical time series for sample 1970-2005.

- Econometric theory tells us that estimating a VAR(1) model by the classical Ordinary Least Squares (OLS) technique on the 1970-2005 time series should do the job.

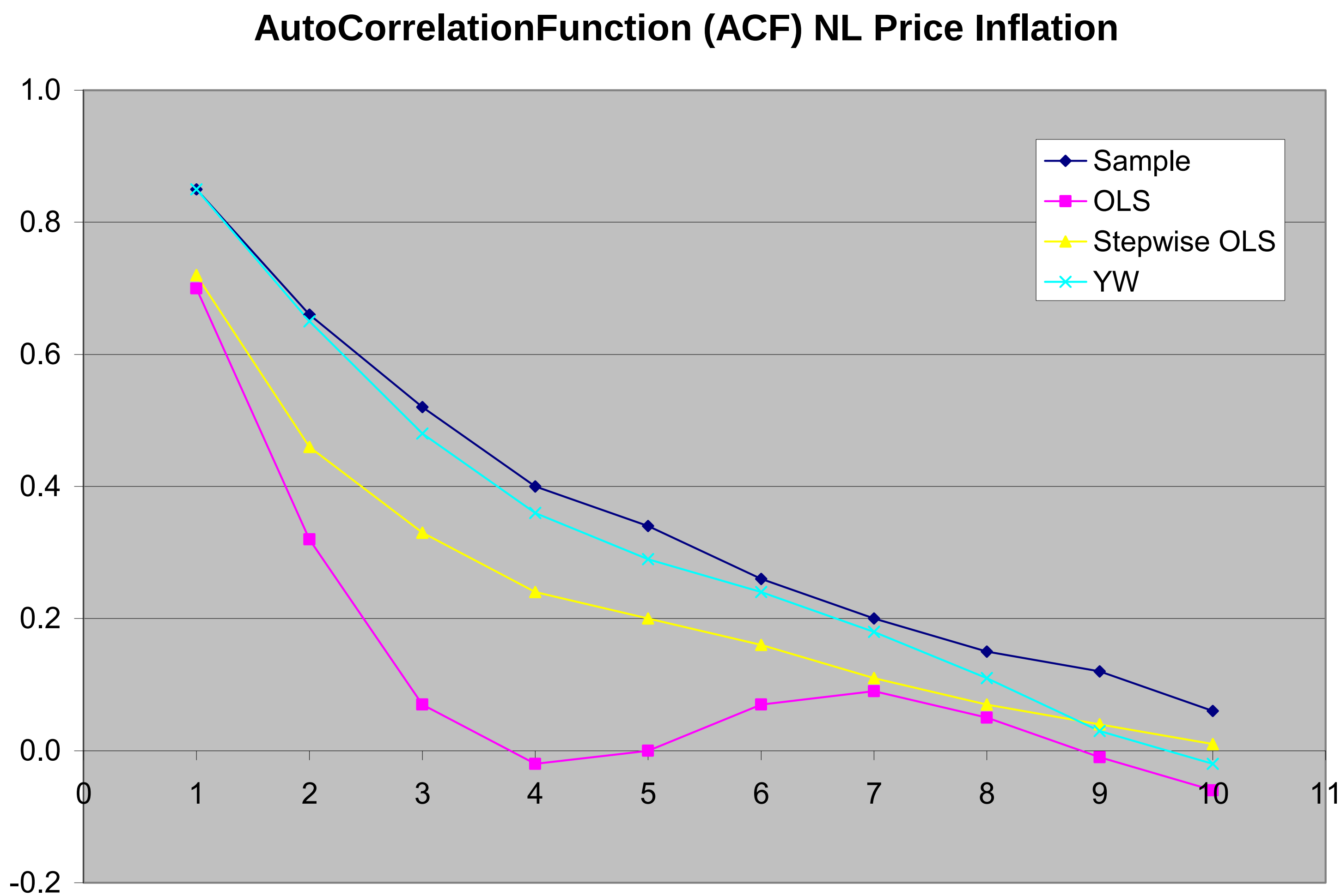
- However, when estimating 315 parameters on 36 observations this need not automatically be the case as is shown by the following correlations and autocorrelations.

Correlations

Correlation differences Model - Sample (OLS)	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
1. Price Inflation NL														
2. Wage Inflation NL	-0.2													
3. Equities EUR return	0.1	0.2												
4. Equities US return	0.2	0.2	0.0											
5. Equities JPN return	-0.3	-0.4	0.1	0.1										
6. Real Estate EUR return	-0.3	-0.2	0.2	0.1	-0.2									
7. USD / EUR return	0.3	0.3	0.0	0.0	0.1	0.1								
8. JPY / EUR return	0.0	0.0	0.0	0.0	0.0	0.1	0.0							
9. Long EUR Interest Rate	0.0	-0.2	0.1	0.1	-0.1	-0.2	0.1	0.0						
10. Short EUR Interest Rate	-0.1	-0.2	0.0	0.0	-0.1	-0.1	0.1	-0.1	-0.1					
11. Long US Interest Rate	0.1	0.0	0.0	0.0	0.0	0.0	0.0	-0.1	0.0	-0.1				
12. Short US Interest Rate	0.1	0.0	0.0	0.0	0.0	0.1	0.0	-0.1	0.0	-0.1	-0.1			
13. Long JPN Interest Rate	-0.2	-0.3	0.1	0.2	-0.2	-0.3	0.2	0.0	0.0	-0.1	0.1	0.1		
14. Short JPN Interest Rate	-0.2	-0.3	0.1	0.1	-0.2	-0.2	0.1	0.0	-0.1	-0.1	0.0	0.0	0.0	

- Table shows differences between (contemporaneous) correlations described by the estimated VAR model and those directly estimated from the sample.
- Given the potential sensitivities shown by the ALM example, each of these differences might as well have a large impact on policy decisions.

Dynamics



- Also the autocorrelations (dynamics) described by the estimated VAR model can be very different then those directly estimated from the sample.

Do's and Don'ts - II

- Look beyond (conditional) variances.
  - Also the dynamics within these variances matter (a lot).

- Make a distinction between true correlations and phase shifts in time.
  - Many economic variables are highly correlated but merely out of phase.
  - ➔ *Example 2 (Appendix)*

- Also consider VAR model orders higher than $p=1$.
  - Especially for low dimensional models $p=1$ is often too restrictive.
  - Use order selection criteria and study the dynamic behavior.

- Avoid automated model selection procedures.
  - These can lead to the wrong parameter restrictions.
  - Differences with sample statistics can get bigger instead of smaller.
  - ➔ *Example 3 (Appendix)*

Do’s and Don'ts - III

- Use (the right) restricted types of VAR models to limit the over parameterization problem
  - Stationarity restriction in the Yule-Walker estimator is a simple example which results in “zero” differences between sample and model statistics.

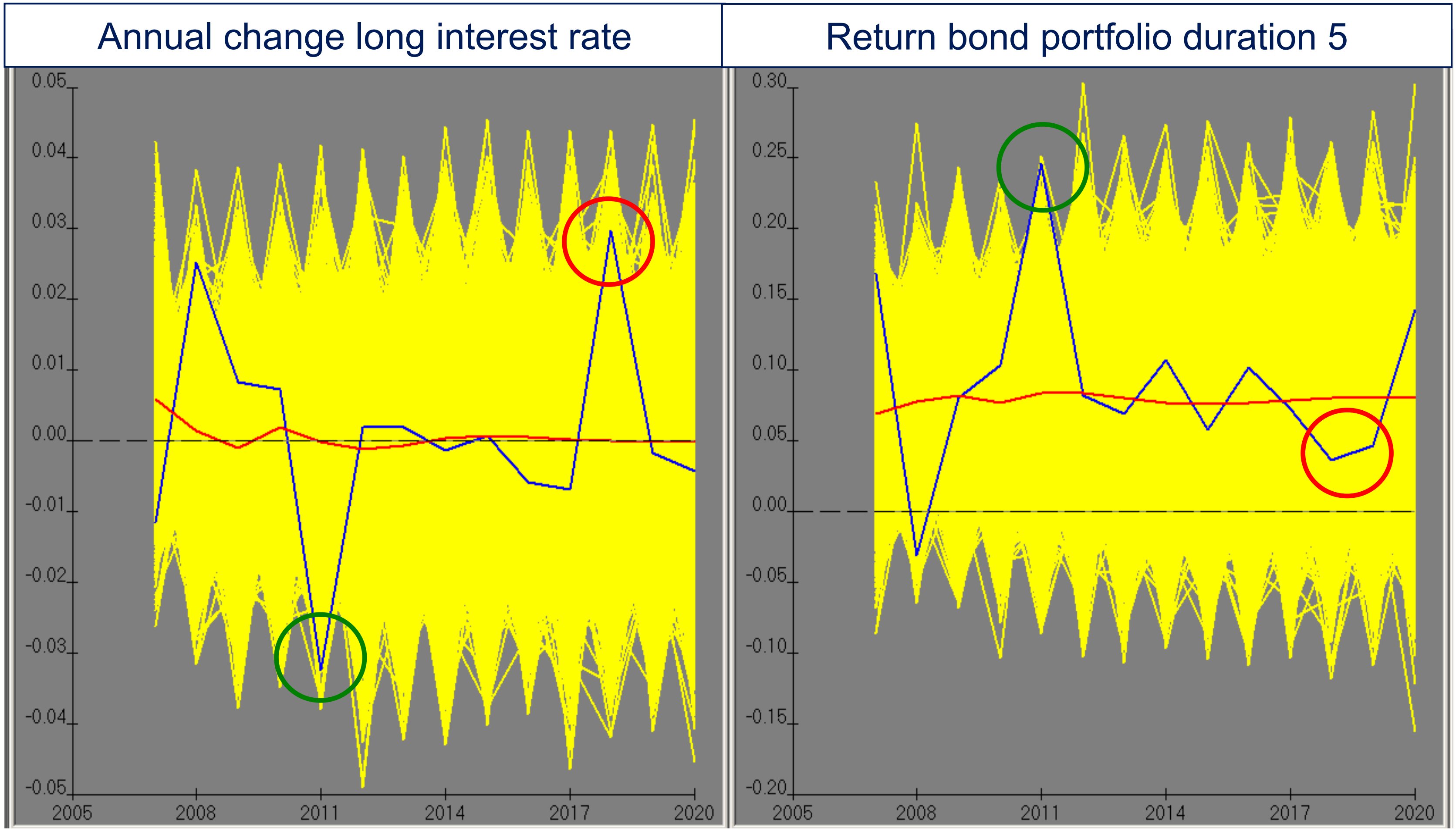
  ➔ *Example 4 (Appendix)*

- Use simple univariate AR models as a reference.
  - Tests show that VAR models are not very good in univariate dynamics.

- Don’t model bond returns but yield curves and cash flows instead.
  - Potential inconsistencies between yield curve changes and bond returns
  - Valuation of pension and insurance liabilities
  - Analysis of duration (matching) policies

  ➔ *Example 5*

Example 5: Yield curves and bond returns



- Historical (duration 5) bond returns approximated as $r(t) \approx r(t-1) - 5\times(r(t) - r(t-1))$
- VAR model from example 1 extended with these bond returns
- Sampling from a (non-structural) stochastic (VAR) model can cause inconsistent scenarios of interest rate (changes) and bond returns.

Do's and Don'ts - IV

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- Don't model individual interest rates.
  - Too many dimensions, especially multi-currency
  - Potential inconsistent yield curves with wrong dynamics
  - Consider parsimonious combination of VAR and Nelson & Siegel model.

  ➔ *Example 6*

- Beware of the symmetry and thin tails of the Normal distribution.
  - Standard VAR model can result in lots of negative interest or inflation rates.
  - Use truncation, logarithmic transformations or other distributions

  ➔ *Example 7 (Appendix)*

- Use other models for risk neutral valuation scenarios.
  - No risk premiums – should exist in real world VAR scenarios
  - Follow the forwards – does not hold empirically
  - Convexity corrections – only a valuation requirement
  - Market snapshot – more is needed for long term decision making

Example 6: VAR and Nelson & Siegel

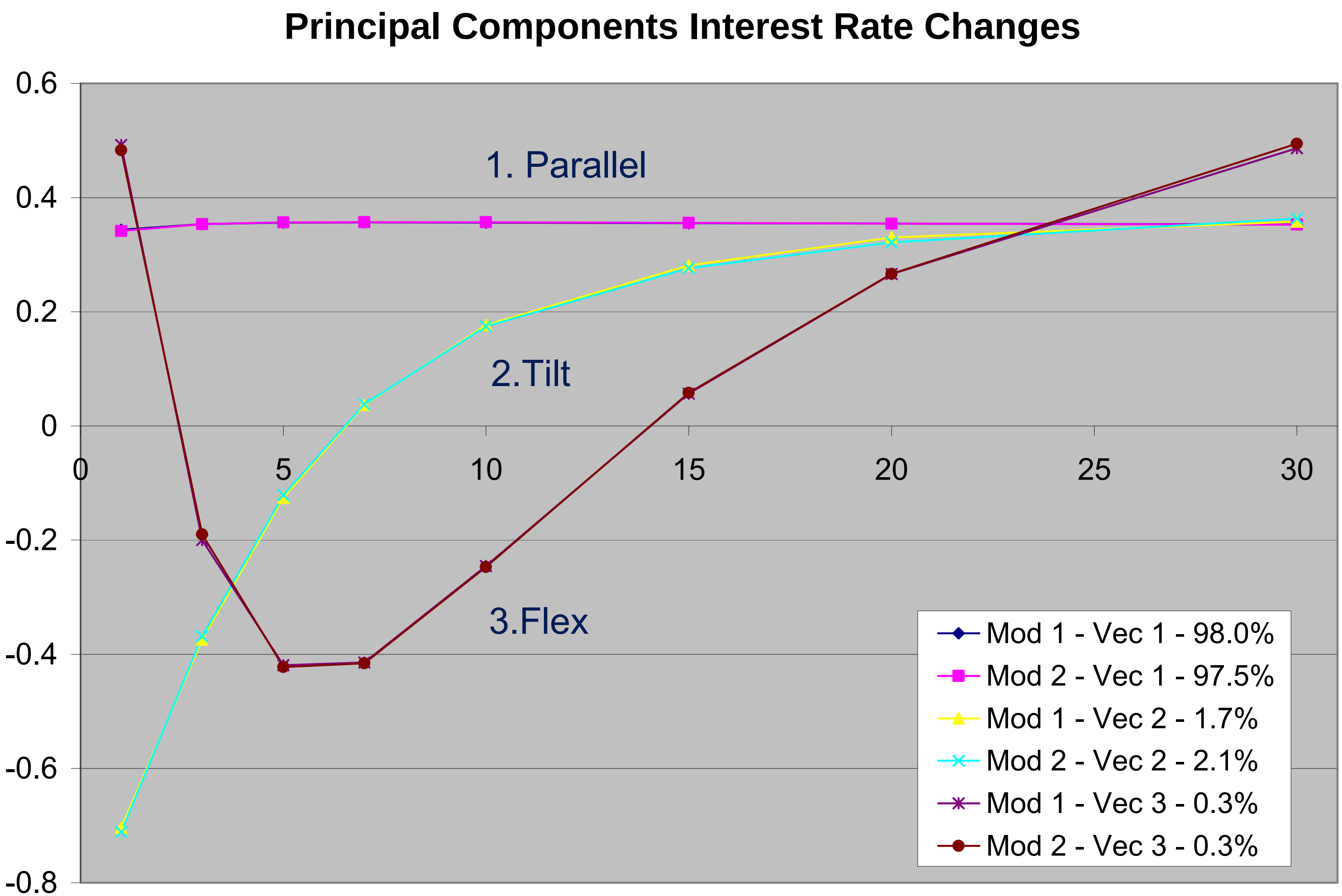
- Generate scenarios
  - of Euro zero coupon yield curve with an horizon of 15 years
  - in accordance with (2^{nd} moment) statistics of historical time series for sample 1979-2005

- Two alternative VAR models
  - 1) Estimate 8 dimensional VAR(1) model on the individual 1, 3, 5, 7, 10, 15, 20 and 30 year yields.
  - 2) First estimate Nelson & Siegel yield curve parameters (β_0, β_1, β_2 and τ) for the sample and then estimate a 3 dimensional VAR(1) model on the β_0, β_1 and β_2 parameters with a fixed value for τ.

$$r(t) = \beta_0 + (\beta_1 + \beta_2)\frac{(1 - e^{-t/\tau})}{(t/\tau)} - \beta_2 e^{-t/\tau}$$

- Model 2 is much more parsimonious than model 1 but describes the same yield curve dynamics as the next slide shows.

Interest Rate Dynamics



Do’s and Don'ts - V

- Don’t go blind on statistical testing and goodness of fit criteria.
  - Assumptions of testing procedures are not always valid in practice, for example asymptotic results versus limited data
  - Combine with (empirical) knowledge of what you want to model
  - Good short term prediction ≠ Good long term scenarios

  ➔ *Example 8 (Appendix)*

- Don’t forget about the low frequencies / long term fluctuations.
  - First order differencing operator (returns) suppresses the low frequencies
  - What matters for long term ALM are the (log) indices and correlations in these terms, not the short term (annual, high frequency) fluctuations

- Never ever just generate “some random scenarios” and then proceed with a detailed subsequent analysis!!
  - Impact of scenario properties can be enormous so proceed with great care
  - There is a lot of (empirical) structure to consider in economics

Want to know more?

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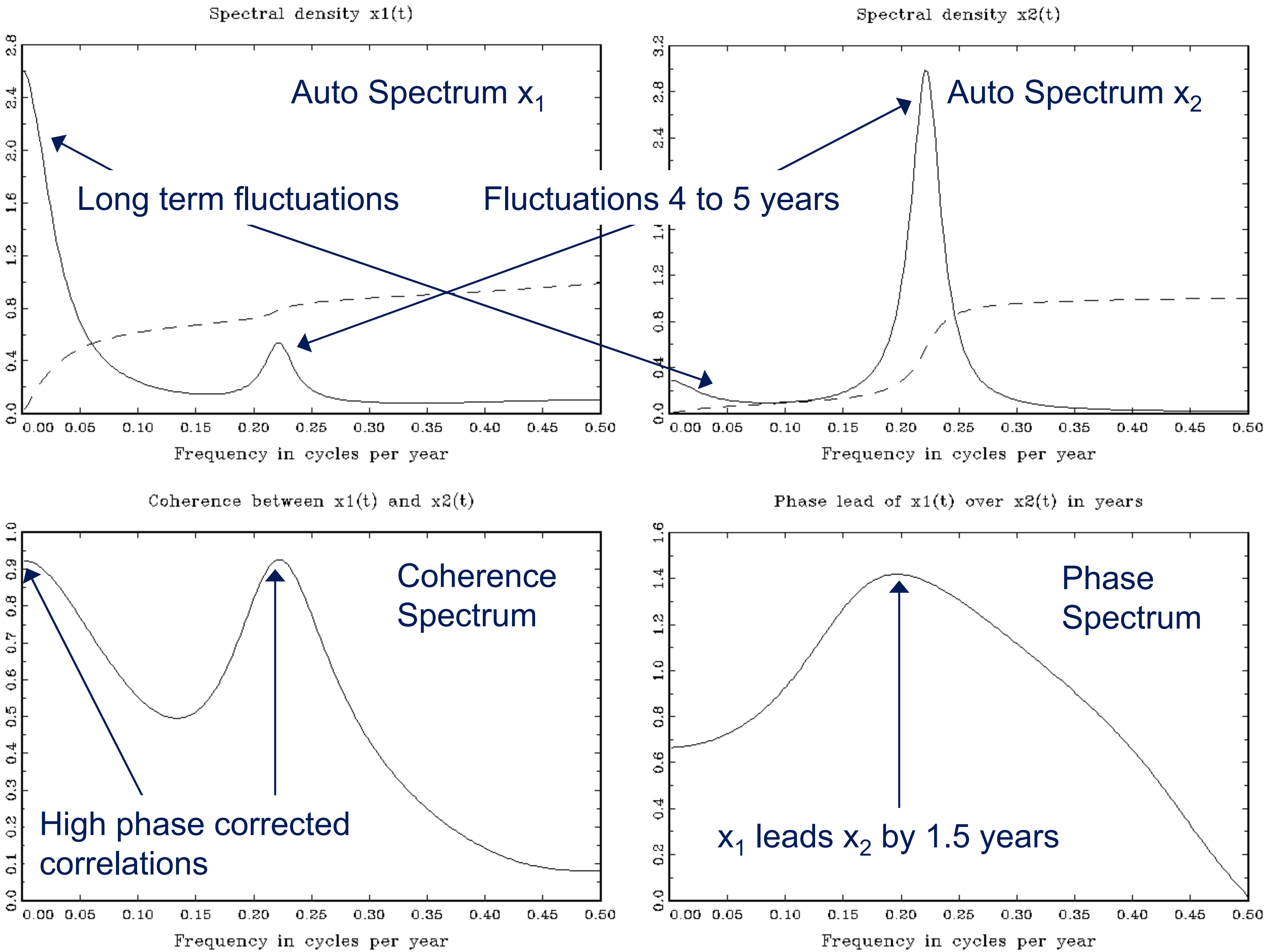
- Steehouwer, H. (2006), "Taking Cycles into Account", *IPE*, June 2006.

Appendix

Multivariate spectral density

2 dimensional VAR(2) model slide 14

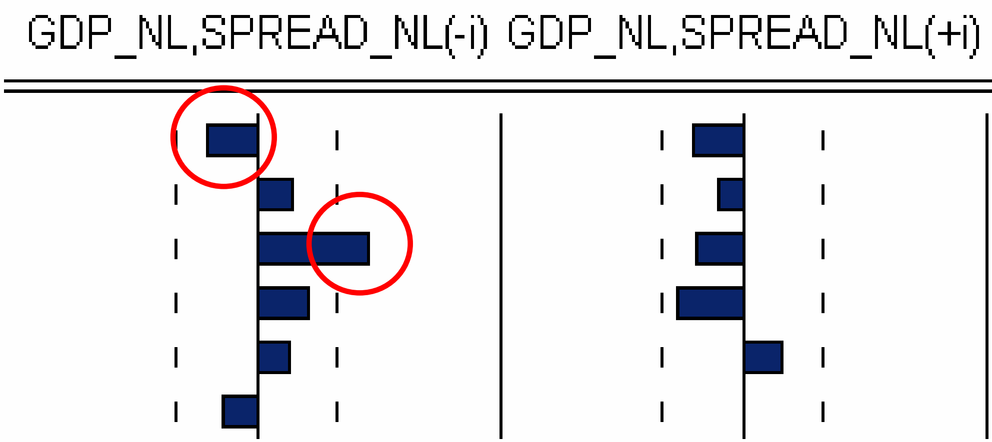


Example 2: Correlations and phase shifts

Do’s and Don’ts slide 20

Cross Correlogram of GDP_NL and SPREAD_NL

Date: 09/11/06 Time: 14:59
Sample: 1970 2004
Included observations: 35
Correlations are asymptotically consistent approximations



i	lag	lead
0	-0.2125	-0.2125
1	0.1614	-0.1005
2	0.4737	-0.1979
3	0.2220	-0.2774
4	0.1327	0.1721
5	-0.1420	0.0014

- Contemporaneous correlation is negative and seems insignificant.
- However correlation between *Spread(t-2)* and *GDP Growth (t)* is high, positive and significant.
- Many economic variables are highly correlated but merely out of phase. Here Spread leads GDP Growth by 2 years.

Example 3: Automated model selection

Do's and Don'ts slide 20

Correlation differences Model - Sample (Stepwise)	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
1. Price Inflation NL														
2. Wage Inflation NL	-0.1													
3. Equities EUR return	-0.1	-0.1												
4. Equities US return	0.0	-0.1	-0.2											
5. Equities JPN return	-0.1	-0.3	0.1	0.0										
6. Real Estate EUR return	-0.1	-0.1	0.1	0.0	-0.2									
7. USD / EUR return	0.1	0.3	-0.2	-0.2	-0.2	0.4								
8. JPY / EUR return	0.1	0.1	-0.1	0.0	0.0	0.1	-0.1							
9. Long EUR Interest Rate	-0.2	-0.4	0.0	0.2	0.1	-0.3	-0.3	0.1						
10. Short EUR Interest Rate	-0.3	-0.3	-0.1	0.2	0.1	-0.4	-0.2	0.0	-0.2					
11. Long US Interest Rate	-0.1	-0.3	0.1	0.2	0.3	-0.1	-0.4	-0.1	0.0	-0.1				
12. Short US Interest Rate	-0.2	-0.3	0.2	0.2	0.3	-0.1	-0.3	-0.1	-0.1	-0.2	0.0			
13. Long JPN Interest Rate	-0.3	-0.4	-0.1	0.2	0.0	-0.4	-0.2	0.0	0.0	0.0	0.0	-0.1		
14. Short JPN Interest Rate	-0.3	-0.4	-0.2	0.2	0.0	-0.4	-0.1	0.0	-0.1	0.0	0.0	-0.2	0.0	

- Same as example 1, but now estimated with Stepwise-OLS.
- Each time restrict the least significant parameter to zero until all remaining parameters are significant at the 95% confidence level (*t*-value 1.64).
- Differences with sample correlations get bigger instead of smaller when compared to the unrestricted OLS estimates (ACF has improved).
- So, apparently did not result in the restrictions we are looking for.

Example 4: Yule Walker estimator

Do's and Don'ts slide 21

- Procedure
  - Estimate autocovariance matrices (covariance, correlations and cross-correlations) until order *p* from sample data.
  - Calculate VAR(*p*) model parameters by solving Yule-Walker equations.
  - By using "biased" sample estimators (divide by T for all lags) a stationarity restriction is imposed on the model which is valuable in case of small samples and many parameters.
  - Asymptotically, OLS and YW estimators are identical.

- Results
  - Until order *p* all model autocovariances will be identical to the sample estimates (ACF has improved).
  - Higher order autocovariances are "extrapolated".
  - So, unlike with the OLS and Stepwise-OLS estimates, now "zero" differences between sample and model (2^{nd} moment) statistics

Example 7: Normal distribution

Do’s and Don’ts slide 23

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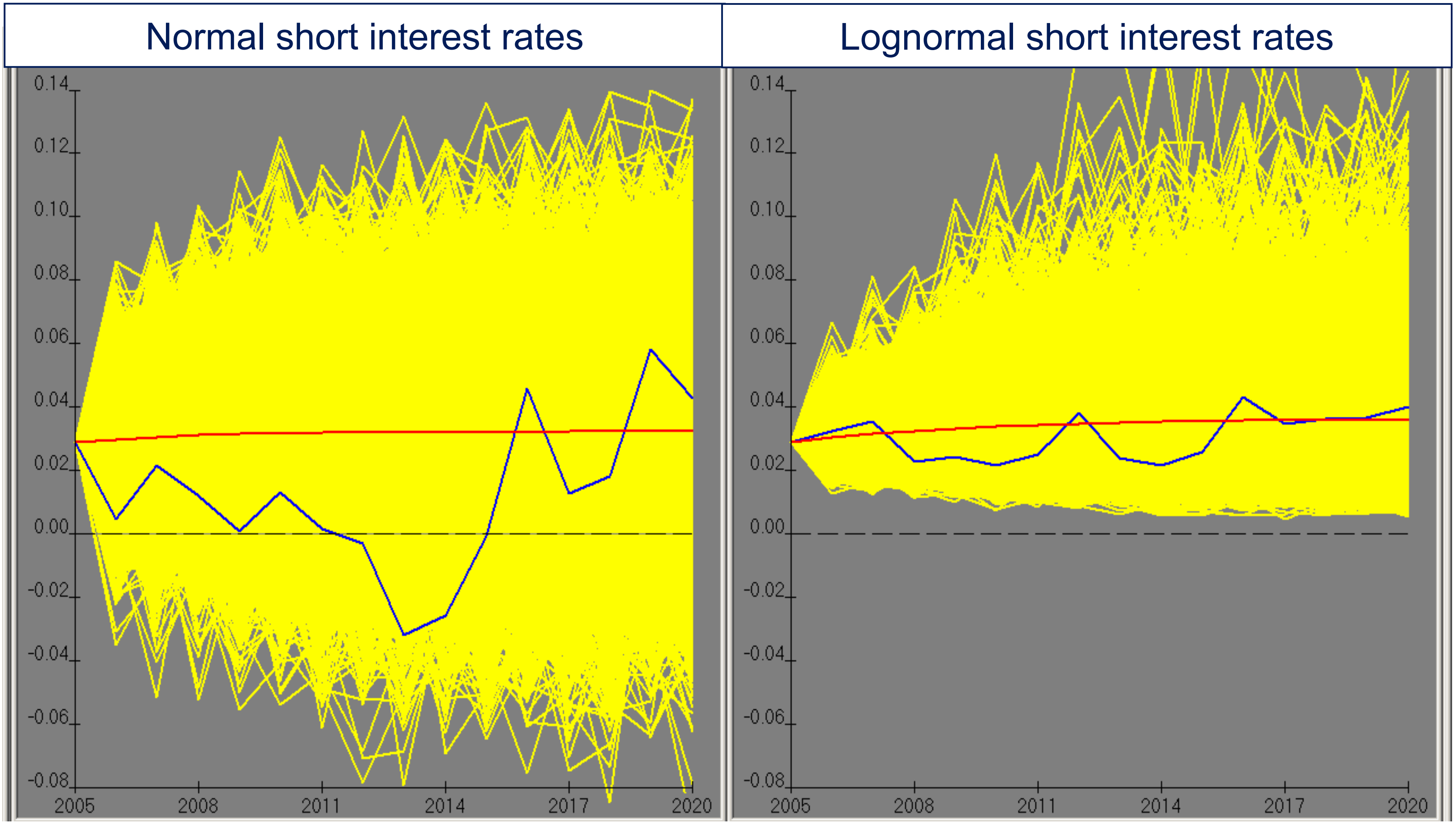

- Standard Normally distributed VAR model can have large probabilities of negative interest or inflation rates.
- Especially for large volatilities and when starting from low initial values.
- A simple logarithmic transformation can be applied to prevent negative values.

Example 8: Statistical testing

Do's and Don'ts slide 26

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- In their famous article Nelson and Plosser (1982) found that, based on the Box and Jenkins identification methodology and statistical testing procedures for unit roots, a so called ARIMA(0,1,1) model to fit most macroeconomic time series well.
  - $x(t)=x(t-1)+\varepsilon(t)+\alpha\times\varepsilon(t-1)$ where $\varepsilon(t)\sim N(0,\sigma^2)$
- In the light of the immense literature on business cycles, it is surprising to say the least that such a model is not able to describe any real cyclical characteristics at all. A simple AR(2)= ARIMA(2,0,0) model may also be consistent with the observed autocorrelations but is also capable of describing (pseudo) cyclical processes.
- This at least warns us not solely to look at the outcome of formal statistical testing procedures but to use other information as well when looking for an appropriate (scenario) model.